An Agenda-Setting Theory of Electoral Competition

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Abstract

The strategy of candidates regarding which policy issues to emphasize during electoral campaigns is an important aspect of electoral competition. In this paper, we advance research on electoral competition by developing a multidimensional model of electoral competition in which parties compete for electoral support by raising the electoral salience of various positional issues. We show that parties have incentive to advertise an issue on which the opponent has electoral advantage or an issue on which neither party has electoral advantage. We also show that the party with the lower equilibrium vote share prefers to emphasize issues on which voters are more ideologically heterogenous, while the party with the higher equilibrium vote share prefers to emphasize issues on which voters are more ideologically homogenous. The analysis provides a theoretical foundation for moving toward a more complete understanding of the content of campaign communication on issues on which voters disagree about which policies ought to be implemented. It also provides novel empirical predictions about how the structure of public opinion impacts the campaign strategy of parties, which can foster further empirical research on electoral campaigns and issue-selection.

Keywords: electoral competition; agenda-setting; issue selection; positional issues.

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The strategy of candidates regarding which policy issues to emphasize during electoral campaigns is an important aspect of electoral competition. An extensive literature documents that candidates selectively emphasize various policy issues in order to sway citizens to put more weight on those considerations when casting their votes (Iyengar and Kinder 1987; Riker 1996; Druckman, Jacobs and Ostermeier 2004). As Stokes (1963) noted that "[t]he skills of political leaders...consist partly in knowing what issue dimensions...can be made salient by suitable propaganda." Such issue-selection strategizing has been widely documented in numerous electoral contests in a variety of countries, including United States, Canada, United Kingdom, Spain, Germany, and Japan, among others (Budge and Farlie 1983; Laver and Hunt 1992; Aldrich and Griffin 2003; McCombs 2004; Druckman, Kifer, and Parkin 2009).

This aspect of electoral competition in which candidates seek to create the most effective campaign strategy by selectively emphasizing various policy issues has been an important topic for research on party competition. Since Donald Stokes (1963) introduced the distinction between "valence issues" and "positional issues," scholars have made important theoretical and empirical advances in our understanding of the determinants of issue-selection mostly in the context of valence issues,[1] issues on which there is a consensus about which policies are desirable and thus the electoral competition is mainly about which candidate is better to deliver what everyone wants (Robertson 1976; Petrocik 1996, Budge and Farlie 1983, Clarke et al. 2009; Egan 2013; Aragones, Castanheira, and Giani 2015, among others). As scholars have argued, in reality, political issues have both valence and positional aspects (Stokes 1963; Miller and Shanks 1996; Clarke et al 2009; Egan 2013)[2], however, the strategy of parties regarding which positional issues to emphasize in electoral campaigns is relatively understudied. For example, we know little about how the structure of public opinion on

[1]In contrast, positional issues are defined as issues on which voters disagree about which policies ought to be implemented.

[2]For example, voters likely agree that low crime is a desirable policy objective, however, they might have different opinions whether this objective can be better achieved by being tough on crime (i.e., imposing harsher penalties) or by addressing socio-economic inequalities.

various policy issues shapes the issue-selection strategy of candidates: Do parties have more incentive to publicize a policy issue on which voters are more ideologically heterogenous or ideologically homogenous? Does a party have incentive to advertise issues on which its opponent has electoral advantage or on which neither candidate has electoral advantage? What positional issues are more likely to be bundled together on a party's electoral agenda?

To answer these questions, we develop a multidimensional model of electoral competition in which parties compete for electoral support by raising the electoral salience of various positional issues. In our framework, the players are two political parties and a continuum of voters; the parties and the voters have ideal policies in an n-dimensional policy space, and the distribution of voters' ideal policies on the n policy issues is multivariate normal. Parties have fixed policy positions and compete to determine which issues are electorally important: each party chooses a vector of advertisement to raise the electoral salience of various issue dimensions in order to maximize its vote share minus the cost of issue advertisement. Each voter elects the party that is closer to his/her policy position on the n issues; the proximity between a voter's and a party's policy position on the n issues is an aggregate of the difference between the voter's and the party's preferred policy on each issue, weighted by the salience of each issue. The relative salience of each issue is endogenously determined by the parties' campaign advertisement choices, and the salience vector determines the distribution of the electorate's preference regarding which party is more electorally desirable.

One might expect that parties choose to increase the salience of issues on which they have electoral advantage (i.e., on which a majority of voters prefer their policy position) because, ceteris paribus, such a strategy augments a party's overall electoral popularity. This is indeed an important consideration of a party's strategic calculus because such advertisement increases a party's vote share by making the center of the electorate relatively closer to that party on the n-policy issue space. However, increasing the salience of an issue also affects how concentrated/dispersed the voters' preferences are on the n policy issues; we label this latter determinant of a party's vote share as the voters' disagreement regarding which party is more desirable on the n policy issues.[3] In other words, when a party chooses which issues to advertise to maximize its vote share, its issue-selection strategy simultaneously impacts the equilibrium vote share through two channels: it changes a party's electoral popularity and it changes the voters' disagreement regarding which party is more desirable on the n policy issues. That voters' disagreement regarding which party is more desirable is an important factor for understanding the issue-selection strategy of parties is missing from the existing literature. Accounting for this factor allows us to uncover new theoretical results regarding the issue-selection strategy of parties in electoral contests.

For example, we show that parties have incentives to advertise an issue on which the opponent has electoral advantage or an issue on which neither party has electoral advantage because raising the salience of such issues can affect the voters' disagreement regarding which party is more desirable in such a way that the overall effect is to increase that respective party's equilibrium vote share. This result underscores some limitations of Riker's influential analysis of issue-selection in electoral contests. Riker (1996) proposes two general principles of electoral campaigns: the dominance principle (a party does not advertise an issue on which the opponent has advantage) and the dispersion principle (if neither of the two parties has advantage on an issue, both parties ignore that issue). Our analysis suggests that these principles need not hold when one scrutinizes the mechanisms by which issue-selection affects a party's vote share.

We also show that accounting for how raising the salience of issues affects voters' disagreement regarding which party is more desirable leads to novel results regarding what kinds of issues a party is more likely to emphasize on its electoral agenda. In particular, we show that the advertisement pattern of the minority party, the party with the lower equilibrium vote share, differs from the advertisement pattern of the majority party, the party with the higher equilibrium vote share, when we consider whether a party is more likely to

[3] Voters' disagreement regarding which party is more desirable depends on both the variance of the voters' ideal policies and the difference between the two parties' policy positions on each issue. We will define this concept precisely in Section 2.

advertise issues on which voters are more ideologically heterogenous or ideologically homogenous. This result expands our understanding of how the strategies of political losers and winners shape the composition of policy agenda in electoral campaigns. Research on issue evolution and manipulation suggests that minority parties have incentive to publicize issues that are more likely to split the majority party's electoral coalition, whereas the majority party has incentive to keep such issues off the electoral agenda (Key 1955; Schattschneider 1960; Carmines and Stimson 1989). This literature focuses on the dynamics of electoral coalitions; it essentially underscores variations in the issues advertised by one party relative to the issues advertised by the other party. Our result is substantively different in that our analysis underscores variations in advertisement among the issues that a party emphasizes *on its electoral agenda*. That is, the minority party puts more advertisement on ideologically heterogenous issues (within the set of positional issues on which the minority party campaigns), and the majority party puts more emphasis on ideologically homogenous issues (within the set of positional issues on the majority party's electoral agenda). These findings provide novel substantive insights into how the strategies of parties shape the composition of policy agenda during electoral contests.

This article adds to a political economy literature on electoral competition. In the classic spatial approach to electoral competition, parties propose policy positions to maximize their electoral success while voters choose the party closest to their policy preferences (Downs 1957). Scholars have complemented this spatial analysis of electoral competition by analyzing other important facets of electoral competition, including the effect of candidates' non-policy characteristics on electoral competition (Groseclose 2001; Krasa and Polborn 2010; Forand 2014), the possibility of strategic entry by new parties (Palfrey 1984; Callander 2005), the effect of negative campaigning on electoral decisions (Skaperdas and Grofman 1995; Polborn and Yi 2006), the influence of party activists and campaign spending in elections (Baron 1994; Grossman and Helpman 1996; Fox and Rothenberg 2011), and the effects of alternative electoral systems on voter choice and party competition (Cox 1987), among other topics. However, the question of issue selection/emphasis is under-researched. This is problematic because, when we think about real elections, an important aspect of electoral competition – encapsulated in the perennial question: "what was this election about?"– is what policy issues to emphasize and what policy issues to ignore. Our paper provides a theoretical foundation for moving toward a more complete understanding of the content of campaign communication on issues on which voters disagree about which policies ought to be implemented.

The paper fits with a scattering of papers on how the electoral agenda is endogenously formed as the result of the strategic competition among political parties (Riker 1986, 1996). The formal literature has analyzed the conditions under which certain policy issues remain on the electoral agenda (Glazer and Lohmann 1999), the conditions under which parties put forward new policy issues relative to the existing status-quo (Colomer and Llavador 2008), the effect of media bias on the parties' incentives to publicize policy issues (Puglisi 2004), the conditions under which candidates emphasize valance issues on which they have an ex-ante advantage when issue ownership is endogenously determined (Aragones, Castanheira, and Giani 2015), whether or not parties emphasize similar issues during electoral campaigns (Hammond and Humes 1995; Simon 2002), and the manipulation of issue dimensions (Moser, Patty, and Penn 2009).[4] Relative to existing formal models that investigate electoral competition through issue selection, our paper imparts several new theoretical results regarding the issue-selection strategies of candidates in the context of positional issues. Our analysis shows a novel mechanism by which the issue advertisement impacts a party's vote share: increasing the salience of an issue affects the voters' disagreement regarding which party is more desirable. This mechanism, to the best of our knowledge, does not appear in any existing formal models of electoral competition through issue selection,[5] and, as we shall

[4]Other papers related to the literature on agenda-setting in elections are: Morelli and van Weelden (2011), which studies how the choice of an (common value or divisive) issue on which to campaign can serve as a credible signal of an incumbent politicians type, and Egorov (2012), which studies a model of election with two candidates and two valence dimensions, where the candidates' competence on each issue is an unobservable random variable.

[5]To uncover the effects of this mechanism it is necessary to work with an n-dimensional policy space,

show, accounting for this mechanism is important for documenting new theoretical results regarding the issue-selection strategy of parties.

Furthermore, the analysis has several empirical implications for the study of electoral campaigns. It shows that voters' disagreement regarding which party is more desirable is an important determinant for understanding patterns of issue-selection in the context of positional issues. Accounting for this factor allows us to assess counterfactuals pertaining to which positional issues parties are more likely to engage if those issues differ in terms of the electoral advantage of parties, the ideological heterogeneity of voters, and the divergence between the parties' policy positions. To that end, the analysis suggests that the minority party is more likely than the majority party to advertise issues on which the opponent has electoral advantage or issues on which neither party has electoral advantage. It also indicates that issues on which there is less ideological heterogeneity among voters are more likely to be advertised as compared to issues on which there is less ideological heterogeneity between parties. The paper also shows that the minority party is more likely to emphasize electoral issues on which voters are more ideologically heterogenous, among those positional issues that form the minority party's electoral agenda, whereas the majority party is more likely to advertise electoral issues on which voters are more ideologically homogenous, among those positional issues that form the majority party's electoral agenda. The analysis provides novel empirical predictions about how the structure of public opinion impacts the campaigning strategy of candidates, which can foster further empirical research on electoral campaigns and issue-selection.

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which further differentiates our model from the few existing formal models of (positional) issue selection (e.g., Simon 2002; Egan 2013). These models typically analyze electoral competition through issue selection in which there is a representative voter (Simon 2002; Egan 2013) or in a two-policy issue space where the total amount of resources allocated to increase the salience of each issue needs to satisfy a budget constraint (Simon 2002; Amoros and Puy 2013). Given this budget constraint, the two-issue electoral competition reduces to a one dimensional model. Questions such as whether ideologically heterogenous or ideologically homogenous policy issues (within the set of potential issues a party may advertise) are more likely to be on the electoral agenda of a party are not addressed by these existing models.

1 Model

The players are a continuum of voters, whose measure is normalized to 1, and two political parties, A and B. The policy space is multidimensional: there are n issue dimensions and the set of possible policy choices for each issue i ($i = 1, 2, ..., n$) is $\mathbf{R}$.

Each party $k \in \{A, B\}$ has an ideal policy, a vector $\mathbf{p}^k = (p_1^k, p_2^k, ..., p_n^k) \in \mathbf{R}^n$, where the i-th element denotes party k's preferred policy on issue i. The parties' most preferred policies on each issue dimension differ; that is, $p_i^A \neq p_i^B$ for all i. The focus of our model is to analyze how parties compete for votes by raising the salience of various issues, and thus we assume the parties' policy positions to be fixed for the duration of the campaign.[6]

Each voter has an ideal policy vector $\mathbf{x} \in \mathbf{R}^n$. The location of the voters' ideal policies follows a multivariate normal distribution, and the voters' positions on various issue dimensions are uncorrelated. That is, a generic voter's ideal policy is $\mathbf{x}_{n\times 1} \sim N(\boldsymbol{\mu}_{n\times 1}, \boldsymbol{\Sigma}_{n\times 1})$ with $\sigma_{ij} = 0$ for any $i \neq j$, where σ_{ij} is the (i, j)-th element of $\boldsymbol{\Sigma}$. We focus on the situation in which voters' policy positions are uncorrelated because it allows us to investigate the effect of the structure of various policy issues on parties' campaign strategies in a simple manner. The assumption is consistent with existing empirical evidence. Since Converse (1964) famously documented that the public shows very little ideological constraint across different policy issues, the empirical evidence has supported the finding of low ideological consistency across separate policy issues (Kinder and Sears 1985; Zaller 1992; Fiorina, Abrams and Pope 2005, among others). Low ideological consistency, of course, is more likely to be the case among those voters that do not have partisan attachments and thus could be persuaded by the parties' campaign messages, the electorate which is the focus of our model, as discussed momentarily. Moreover, in an extension of the model, we also analyze the situation in which voter preferences across issues are correlated (i.e., $\sigma_{ij} \neq 0$ for $i \neq j$) to show that the main

[6] Fixed policy positions can be due to a previous electoral stage that is not modeled here. The existing theoretical literature suggests several factors as to why the policy platforms of parties differ when they enter the general electoral competition, including the influence of party activists (Baron 1994) and the influence of internal bargaining and nomination processes (Coleman 1971), among other factors.

results are robust to this extension.

In our framework, each issue dimension can thus be characterized in terms of (μ_i, σ_{ii}), the mean and the variance of the distribution of voters' ideal policies on policy issue i, where σ_{ii} is the i-th diagonal element of $\mathbf{\Sigma}$. Thus the pair (μ_i, σ_{ii}) can be thought as characterizing the existing public opinion on policy issue i.

Given the existing public opinion on various policy issues, each party $k \in \{A, B\}$ chooses an amount of advertisement for each issue dimension, a vector $\mathbf{a}^k = (a_1^k, a_2^k, ..., a_n^k) \in \mathbf{R}_+^n$ at a cost $C^k(\mathbf{a}^k) = \sum_{i=1}^n c^k(a_i^k)$. The campaign advertisement can be thought as the amount of money, time, and effort parties allocate to emphasize certain policy issues during electoral campaigns in order to persuade voters that those issues are a governing priority. We assume that the cost function is twice continuously differentiable, $c^{k'}(\cdot) > 0$, $c^{k''}(\cdot) > 0$, $c^k(0) = 0$, $c^{k'}(0) = 0$, $\mathbf{lim}_{a^k \to \infty} c^k(a^k) = \infty$ and $\mathbf{lim}_{a^k \to \infty} c^{k'}(a^k) = \infty$ for $k \in \{A, B\}$.[7] The objective of each party is to maximize the vote share less the cost of issue advertising. Thus party k's utility is

$$U_k(\mathbf{a}^k; \mathbf{a}^{-k}) = v^k(\mathbf{a}^k; \mathbf{a}^{-k}) - C^k(\mathbf{a}^k),$$

where $\mathbf{a}^k$ is the vector of advertisement on the n issues by party k and v^k is party k's vote share, which will be characterized in the next section.

A voter's preference over policies depends on the difference between the implemented policy and her ideal policy on each issue and on the relative importance the voter puts on each policy issue. Thus the utility of a voter with ideal policy $\mathbf{x}$ is

$$U_v = -\sum_{i=1}^{n} w_i(\mathbf{a})(p_i - x_i)^2, \tag{1}$$

where $\mathbf{a} = (\mathbf{a}^A, \mathbf{a}^B)$ and $w_i(\mathbf{a})$ represents the relative importance the voter puts on issue

[7]The focus of our model is to analyze the strategy of issue advertisement in campaigns. However, candidates during electoral campaigns devote resources to a host of other activities such as hiring staff, polling, analyzing data, and developing ground organizations that can be crucial for the turnout of voters and core constituency at election day. As such, the cost of issue advertisement can be thought as an opportunity cost.

i for $i = 1, 2, ..., n$. The electoral salience of various policy issues is affected by the parties' issue advertisement. Specifically, for each issue i denote by $a_i = \Sigma_k a_i^k$ the total amount of advertisement issue i receives in an electoral campaign. Because the relative importance of each issue depends on the total advertisement of that respective issue, we can re-express the vector of advertisement on the n issues as $\mathbf{a} = (a_1, a_2, ..., a_n)$. The issue salience vector then is a function of the parties' advertisement strategy: $\mathbf{w}(\mathbf{a}) = \{w_i(\mathbf{a})\}_{i=1}^n$. We assume that $w_i(\mathbf{a}) = \frac{f(a_i)}{\sum_{i=1}^n f(a_i)}$, where $f(\cdot)$ is an increasing continuously differentiable function and $f(x) > 0$ for all x and therefore $\sum_{i=1}^n w_i(\mathbf{a}) = 1$.[8]

The contest success function $w_i(\mathbf{a})$ encapsulates how the parties' campaign advertisement effort to highlight which issues are more important translates into the voters' assessments regarding the relative salience of various policy issues. We take a reduced-form model of this process because our main interest is to investigate the issue-selection incentives of the parties.[9] Our approach here is similar to other papers that use contest functions to model how campaign advertisement and spending affect the behavior of voters (Snyder 1989; Baron 1994; Skaperdas and Grofman 1995; Grossman and Helpman 1996). The contest success function can be derived from axiomatic theories of (partially) uninformed voting (Luce 1959), from an inferential process of an (uninformed) audience that observes evidence produced by contestants who seek to persuade the audience of the correctness of their views (Skaperdas and Vaidya 2012) or from a political contest in which parties provide costly information to voters (Gul and Pesendorfer 2012).

More importantly, the key assumption of contest function $w_i(\mathbf{a})$ that if an issue receives more advertisement than others, the relative electoral importance of that policy issue increases has garnered significant empirical support. An extensive empirical literature documents that the amount of media coverage or candidate discussion of certain policy issues

[8] Examples of such functions are: $w_i(\mathbf{a}) = \frac{a_i+\alpha}{\sum_{i=1}^n a_i + n\alpha}$, $\alpha > 0$; $w_i(\mathbf{a}) = \frac{a_i^2+\alpha}{\sum_{i=1}^n a_i^2 + n\alpha}$, $\alpha > 0$; $w_i(\mathbf{a}) = \frac{e^{a_i}}{\sum_{i=1}^n e^{a_i}}$; and $w_i(\mathbf{a}) = \frac{log(a_i+\alpha)}{\sum_{i=1}^n log(a_i+\alpha)}$, $\alpha > 1$.

[9] In this sense, the paper is related to other models of electoral competition which analyze the strategic behavior of candidates given various notions of voting behavior: for example, Adams (2001), Adams, Merrill III, and Grofman (2005), Callander and Wilson (2008), Bendor et al (2011), and Diermeier and Li (2013).

induces citizens to give more weight to those issues when evaluating candidates (McCombs and Shaw 1972; Iyengar and Kinder 1987; Krosnick and Kinder 1990; Johnston et al 1992; Ansolabehere and Iyengar 1994; Jacobs and Shapiro 1994; Carsey 2000; Simon 2002; Druckman, Jacobs and Ostermeier 2004, Bartels 2006, among others), effects that have been shown both in observational and experimental studies. Moreover, such agenda-setting effects have been documented on a wide array of policy issues, during both national and local elections, in a variety of countries including United States, Spain, Germany, and Japan, among others (McCombs 2004). The model essentially builds upon these well-documented empirical patterns to investigate the issue-selection strategies of candidates in electoral contests.

Of course, not all voters are susceptible to agenda-setting effects. Specifically, some voters cast their votes on the basis of party identification, regardless of the parties' campaign message. That is, such partisan voters have an allegiance for one party or another, and thus campaign messages and advertisements will have little effect on their voting decision. Notice that we could follow a similar modeling strategy as Baron (1994) and Grossman and Helpman (1996) and model the electorate as consisting of both a fraction of voters who are susceptible to campaign effects and a fraction of voters who are not; such modeling would not affect the forthcoming analysis and thus we focus our model on those (non-partisan) voters who can be susceptible to campaign effects.

The game unfolds as follows. In the first stage, the parties simultaneously choose their advertisement strategies regarding which issue dimensions to emphasize. The second stage is a standard voting game: each voter makes a decision regarding which party to elect.

2 Party Competition and Issue Selection

In the voting stage, given the parties' strategies of advertisement and the voters' utility function as defined by expression (1), a voter with ideal policy $\mathbf{x}$ prefers party A over B if and only if

$$\sum_{i=1}^{n} w_i(\mathbf{a})(x_i - p_i^A)^2 < \sum_{i=1}^{n} w_i(\mathbf{a})(x_i - p_i^B)^2,$$

which is equivalent to

$$\sum_{i=1}^{n} w_i(\mathbf{a}) d_i(x_i) > 0, \tag{2}$$

where $d_i(x_i) \equiv (p_i^A - p_i^B)(x_i - \frac{p_i^A + p_i^B}{2})$.

Expression (2) indicates the voters' optimal electoral decision given the advertisement strategy of the parties. A key parameter of (2) is $d_i(x_i)$, which can be thought as a measure of whether and by how much a voter with ideal policy x_i prefers party A or party B on policy issue i. A positive $d_i(x_i)$ means that such a voter derives a higher utility from party A's policy than from party B's policy on issue i, and a larger $d_i(x_i)$ means that the difference between this voter's utility from A's policy and from B's policy is larger. Hence a positive $d_i(x_i)$ implies that a voter with ideal policy x_i prefers party A over party B on issue i, and a larger $d_i(x_i)$ implies that party A has a bigger advantage over party B on issue i for such a voter. Conversely, a negative $d_i(x_i)$ implies that a voter with ideal policy x_i prefers party B over party A on issue i, and a bigger $|d_i(x_i)|$ (i.e. a smaller $d_i(x_i)$) implies that party B has a bigger advantage over party A on issue i.

Given the voters' strategy previously analyzed, the vote share of a party is the fraction of the electorate that prefers that respective party over the other party. For example, party A's vote share is:

$$v^A(\mathbf{a}^A; \mathbf{a}^B) = \mathbf{P}(x| \sum_{i=1}^{n} w_i(\mathbf{a})(x_i - p_i^A)^2 < \sum_{i=1}^{n} w_i(\mathbf{a})(x_i - p_i^B)^2),$$

which is equivalent to

$$v^A(\mathbf{a}^A; \mathbf{a}^B) = \mathbf{P}(\mathbf{x}| \sum_{i=1}^{n} w_i(\mathbf{a}) d_i(x_i) > 0). \tag{3}$$

The vote share of party B can be derived in an analogous manner:

$$v^B(\mathbf{a}^A; \mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\sum_{i=1}^{n} w_i(\mathbf{a})d_i(x_i) < 0). \tag{4}$$

Expressions (3) and (4) indicate that an important determinant of a party's vote share is $\mathbf{d}(\mathbf{x}) \equiv \{d_i(x_i)\}_{i=1}^n$. Hence the vector $\mathbf{d}(\mathbf{x})$ characterizes whether and by how much a voter with ideal policy $\mathbf{x}$ prefers party A over party B on each of the n policy dimensions. Because the distribution of voters' policy positions follows a multivariate normal distribution, i.e. $\mathbf{x} \equiv \{x_i\}_{i=1}^n \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, the vector $\mathbf{d}(\mathbf{x}) = \{(p_i^A - p_i^B)(x_i - \frac{p_i^A + p_i^B}{2})\}_{i=1}^n$ is also multivariate normal:

$$\mathbf{d}(\mathbf{x}) \sim N(\boldsymbol{\nu}^A, \boldsymbol{\Lambda}),$$

where $\nu_i^A = (p_i^A - p_i^B)(\mu_i - \frac{p_i^A + p_i^B}{2})$ is the i-th element of vector $\boldsymbol{\nu}^A$, and $\lambda_{ij} = (p_i^A - p_i^B)(p_j^A - p_j^B)\sigma_{ij}$ is the i,j-th element of matrix $\boldsymbol{\Lambda}$. Furthermore, since $\sigma_{ij} = 0$ for all $i \neq j$, we have $\lambda_{ij} = 0$ for all $i \neq j$. Therefore, the variance-covariance matrix $\boldsymbol{\Lambda}$ can be fully characterized by the diagonal entries $\lambda_{ii} = (p_i^A - p_i^B)^2\sigma_{ii}$. Parameters ν_i^A and λ_{ii} have important implications for our analysis which we will explain shortly in subsequent paragraphs.

Similarly, we define $\nu_i^B \equiv (p_i^B - p_i^A)(\mu_i - \frac{p_i^A + p_i^B}{2}) = -\nu_i^A$, $\boldsymbol{\nu}^B \equiv \{\nu_i^B\}_{i=1}^n$ for party B, and we can use $-\mathbf{d}(\mathbf{x}) = \{(p_i^B - p_i^A)(x_i - \frac{p_i^A + p_i^B}{2})\}_{i=1}^n$ as the measure of whether and by how much a voter with ideal policy $\mathbf{x}$ prefers party B over party A on each of the n policy issues. Similar to the derivation above, we have $-\mathbf{d}(\mathbf{x}) \sim N(\boldsymbol{\nu}^B, \boldsymbol{\Lambda})$. In the subsequent analysis, we use the notation ν_i^k for $k \in \{A, B\}$ and, without loss of generality, denote $\nu_i = \nu_i^A$ where indexing by k is not important.

Therefore, the measure of how much a voter with ideal policy $\mathbf{x}$ prefers party $k \in \{A, B\}$ over the other party on each of the n issues follows a normal distribution $N(\boldsymbol{\nu}^k, \boldsymbol{\Lambda})$. We can then rewrite the two parties' vote shares as follows:

$$v^A(\mathbf{a}^A;\mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\mathbf{w}(\mathbf{a})\cdot\mathbf{d}(\mathbf{x})>0) = \Phi\left(\frac{\sum_{i=1}^n w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^n w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}}\right), \tag{5}$$

and

$$v^B(\mathbf{a}^B;\mathbf{a}^A) = 1 - v^A(\mathbf{a}^A;\mathbf{a}^B)$$

where $\Phi(\cdot)$ is the cdf of standard normal distribution.[10]

As mentioned, the vector $\mathbf{d}(\mathbf{x})$ measures whether a voter with ideal policy $\mathbf{x}$ prefers party A over its opponent on the n policy issues. As such, $\nu_i^A = (p_i^A - p_i^B)(\mu_i - \frac{p_i^A+p_i^B}{2})$ can be interpreted as the difference between the utility from party A's policy on issue i and from party B's policy for a voter with ideal policy μ_i on issue i.

Since μ_i is the center of the distribution of voters' ideal policies on issue i, one can think of the parameter ν_i^A as a measure of party A's electoral popularity on policy issue i. A positive ν_i^A implies that a majority of the voters prefers party A over party B on policy issue i; that is, party A has an advantage on issue i, and a bigger ν_i^A implies a bigger electoral advantage for party A relative to party B on policy issue i. Conversely, a negative ν_i^A (i.e., a positive ν_i^B) implies that a majority of the voters prefers party B over party A on policy issue i; that is, party B has electoral advantage on issue i, and a bigger $-\nu_i^A$ (i.e., a bigger ν_i^B) implies a bigger electoral advantage for party B relative to party A on policy issue i. Moreover, one can think of $\sum_{i=1}^n w_i(\mathbf{a})\nu_i$, the numerator in expression (5), as a measure of party A's electoral popularity on the n policy issues; how a party's electoral popularity is aggregated across the n policy issues is determined by the salience of each issue dimension, $w_i(\mathbf{a})$. Therefore we have the following definition:

[10] The calculation is as follows:

$$v^A(\mathbf{a}^A;\mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\mathbf{w}(\mathbf{a})\cdot\mathbf{d}(\mathbf{x})>0) = \mathbf{P}(\mathbf{x}|\frac{\mathbf{w}(\mathbf{a})\cdot\mathbf{d}(\mathbf{x}) - \mathbf{w}(\mathbf{a})\cdot\boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}} > -\frac{\mathbf{w}(\mathbf{a})\cdot\boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}}) =$$

$$= (1-\Phi\left(-\frac{\mathbf{w}(\mathbf{a})\cdot\boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}})\right) = \Phi\left(\frac{\mathbf{w}(\mathbf{a})\cdot\boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}}\right) = \Phi\left(\frac{\sum_{i=1}^n w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^n w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}}\right).$$

Definition 1. *For $k \in \{A, B\}$, we define party k's electoral popularity on issue i as $\nu_i^k = (p_i^k - p_i^{-k})(\mu_i - \frac{p_i^A + p_i^B}{2})$ and party k's electoral popularity on the n policy issues as $\sum_{i=1}^n w_i(\mathbf{a})\nu_i^k$.*

In the remaining of the paper we will refer to the electoral popularity of a party on the n issues simply as a party's electoral popularity; on the other hand, when we refer to the electoral popularity of a party on issue i we will qualify that statement by referring to the respective issue.

The parameter σ_{ii} measures the variance of voters' ideal positions on issue dimension i, and a larger (smaller) σ_{ii} implies a more heterogenous (more homogenous) electorate on policy issue i.[11] The parameter $\lambda_{ii} = (p_i^A - p_i^B)^2\sigma_{ii}$ is the variance of $d_i(x_i) = (p_i^A - p_i^B)(x_i - \frac{p_i^A + p_i^B}{2})$; notice that λ_{ii} depends on both the variance of voters' preferences, σ_{ii}, and also on the difference between the parties' policy positions, $(p_i^A - p_i^B)^2$. One can think of λ_{ii} as a measure of voters' disagreement regarding which party is more desirable relative to its opponent on policy issue i, and a larger (smaller) λ_{ii} connotes a higher (lower) voters' disagreement regarding which party is more desirable on policy issue i. Moreover, one can think of $[\sum_{i=1}^n w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}$, the denominator in expression (5), as a measure of voters' disagreement regarding which party is more desirable on the n policy issues; how the voters' disagreement regarding which party is more desirable is aggregated across the n issue dimensions is determined by the salience of each issue dimension, $w_i(\mathbf{a})$. Therefore we have the following definition:

Definition 2. *We define the voters' disagreement regarding which party is more desirable on policy issue i as $\lambda_{ii} = (p_i^A - p_i^B)^2\sigma_{ii}$ and the voters' disagreement regarding which party is more desirable on the n policy issues as $[\sum_{i=1}^n w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}$.*

In the remaining of the paper we will refer to the voters' disagreement regarding which party is more desirable than the other party on the n issues simply as the voters' disagreement; on the other hand, when we refer to the voters' disagreement regarding which party is

[11] σ_{ii} is the variance of x_i.

more desirable on issue i we will qualify that statement by referring to the respective issue.

Given the parties' vote shares previously analyzed, we next analyze the parties' optimal advertisement strategies. In this context, party A's optimization problem is

$$\mathbf{max}_{\mathbf{a}^A \in \mathbf{R}^n_+} \Phi\left(\frac{\sum_{i=1}^n w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^n w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}}\right) - C^A(\mathbf{a}^A).$$

Note that the action space in our model is compact, even though we formulate the advertisement of each party as chosen from $\mathbf{R}^n_+$. This is because $\Phi\left(\frac{\sum_{i=1}^n w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^n w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}}\right) \leq 1$ for all $\mathbf{a}^A$, but $C^A(\mathbf{a}^A) = \sum_{i=1}^n c^A(a_i^A)$ and $c^{A'}(\infty) = \infty$. Therefore there exists $\bar{a} > 0$ such that the above optimization problem is equivalent to maximizing the same objective function by choosing $\mathbf{a}^A \in [0, \bar{a}]^n$. The same argument applies for party B's optimization problem below. Party B's optimization problem is

$$\mathbf{max}_{\mathbf{a}^B \in \mathbf{R}^n_+} \left[1 - \Phi\left(\frac{\sum_{i=1}^n w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^n w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}}\right)\right] - C^B(\mathbf{a}^B).$$

In the spatial model of electoral competition, equilibria in pure strategy do not generally exist in a multidimensional policy space because the continuity condition necessary for the existence of such an equilibrium is satisfied only under very restrictive conditions. To overcome the continuity problem, scholars have developed probabilistic voting models, where citizens vote according to probability functions based on their preferences (Coughlin 1992).[12] Our set-up here is similar to probabilistic voting model as the continuity condition is satisfied in our framework because a party's strategy is to choose an amount of advertisement on each issue and each party's utility function is continuous in the advertisement strategies. In the remaining of our analysis we characterize the issue-selection incentives of parties in a pure strategy equilibrium.

First, we show that the parties will not advertise the same policy issue. Intuitively, if

[12] Probabilistic voting models have been developed and analyzed in Hinich, Ledyard and Ordeshook (1973), Coughlin and Nitzan (1981), Enelow and Hinich (1989), and the notion of a "local equilibrium" is used in some of these papers for existence results.

both parties were to advertise some issue i in equilibrium, the optimization problems of parties imply that the vote shares of both parties increase in the advertisement on issue i. However, because increasing one party's vote share implies decreasing the vote share of the other party, both parties' objective functions cannot increase simultaneously in the advertisement on issue i. Thus we have the following result:

Proposition 1. *The two parties will not advertise the same policy issue (that is, $a_i^{A*}a_i^{B*} = 0$ for all $i = 1, 2, ..., n$).*

A policy issue in our framework can be interpreted both as a policy area such as national security or the economy, but also as a dimension within a policy area such as the unemployment or the deficit, both of which are economic issues. Proposition 1 does not exclude the fact that both parties can advertise the same policy area, but suggests that even when parties do so, they will emphasize different considerations of that respective policy domain. For instance, both parties could emphasize the topic of law and order in an electoral campaign, however, Proposition 1 suggests that they will raise the salience of different aspects. The 1992 presidential election offers an example: both parties talked about crime, however, the Republicans emphasized punishment, while the Democrats stressed prevention as the main way of tackling crime (Holian 2004).[13]

We label the party with the higher equilibrium vote share as *the majority party* and the party with the lower equilibrium vote share as *the minority party.*[14]

Definition 3. *The* **majority party** *is the party whose equilibrium vote share is greater than $\frac{1}{2}$ and the* **minority party** *is the party whose equilibrium vote share is less than $\frac{1}{2}$.*

A positive $\sum_{i=1}^{n} w_i(\mathbf{a})\nu_i$, the numerator of expression (5), implies that a majority of the voters prefers party A over party B on the n policy issues, and a bigger $\sum_{i=1}^{n} w_i(\mathbf{a})\nu_i$

[13]For an analysis of such an issue-framing strategy in the context of public spending, see Jacoby (2000).

[14]It is possible that parties can get an equal vote share in equilibrium if, for example, parties' policy positions are symmetric around the mean of the distribution of voters' ideal policies on all issue dimensions, which implies that $\nu_i = 0$ for all issues. In this case, issue-advertisement has no effect on a party's vote share so parties would choose 0 advertisement on all issues since advertisement is costly; thus we focus our analysis on those situations in which issue-selection matters.

implies a bigger electoral advantage for party A relative to party B on the n policy issues. Therefore, party A is the majority party if $\sum_{i=1}^{n} w_i(\mathbf{a})\nu_i > 0$ and party B is the majority party if $\sum_{i=1}^{n} w_i(\mathbf{a})\nu_i < 0$. Whether party A or party B is the majority party depends on the exogenous parameters (e.g., the distribution of voters' ideal policies, the parties' cost functions of issue advertisement, etc). The identity of the majority (or the minority) party is not relevant for the results we state below since we characterize the issue-selection incentives of parties in any pure strategy equilibria, and thus the characterization results hold for all such equilibria. To this end, we use the following definition in the subsequent analysis:

Definition 4. *We denote by I_k the set of issues that party k may advertise; that is $I_k \equiv \{i | a_i^{*-k} = 0\}$.*

Proposition 1 shows that parties do not advertise the same issue in equilibrium. Thus, one can think of I_k as the set of potential issues party k might advertise in an equilibrium.

Before we proceed with analyzing the issue-selection strategies of parties, let us illustrate the two channels by which a party can affect its vote share when advertising various issues. To that end, consider the vote share of party A, given parties' advertisement strategies $\mathbf{w}(\mathbf{a}) = \{w_i(\mathbf{a})\}_{i=1}^{n}$:

$$v^A(\mathbf{a}) = \Phi\left(\frac{\sum_{i=1}^{n} w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^{n} w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}}\right).$$

Naturally, one would expect that party A will advertise those issues on which it has an electoral advantage; that is, party A will advertise issues with $\nu_i > 0$ because such advertisement increases its electoral popularity (e.g. it increases $\Sigma_{i=1}^{n} w_i(\mathbf{a})\nu_i^A$). Emphasizing issues on which it has electoral advantage is an important consideration of party A's strategic calculus but it is not the complete story. There is another channel by which advertising an issue can change party A's equilibrium vote share: emphasizing issue i also affects the voters' disagreement regarding which party is more desirable (e.g. it also affects $[\sum_{i=1}^{n} w_i(\mathbf{a})^2\lambda_{ii}]^{\frac{1}{2}}$).

To illustrate more clearly this mechanism, suppose that $p_i^A = 1/2$ and $p_i^B = -1/2$ for all i, and $\mu_i = \mu \neq 0$ for all i. Recall that the electoral advantage of party A on issue

i is defined as $\nu_i = (p_i^A - p_i^B)(\mu_i - \frac{p_i^A + p_i^B}{2})$ and the voters' disagreement regarding which party is more desirable on issue i is defined as $\lambda_{ii} = (p_i^A - p_i^B)^2 \sigma_{ii}$. Given our parametric specifications, we have $\nu_i = \mu$ for all i and $\lambda_{ii} = \sigma_{ii}$, which implies that party A's vote share is $\Phi\left(\frac{\mu}{[\sum_{i=1}^n w_i(\mathbf{a})^2 \sigma_{ii}]^{\frac{1}{2}}}\right)$. In this scenario, party A cannot affect its vote share by choosing to advertise issues to increase its electoral popularity because the electoral popularity of party A is μ regardless of which issues party A chooses to emphasize. However, party A's issue-selection strategy can influence the voters' disagreement regarding which party is more desirable (i.e., it can affect $[\sum_{i=1}^n w_i(\mathbf{a})^2 \sigma_{ii}]^{\frac{1}{2}}$, and therefore party A will select issues on the basis of σ_{ii} so as to increase its vote share through this channel. A similar reasoning holds for party B in this example.

In general, when a party chooses which issue to advertise, its issue-selection strategy impacts the equilibrium vote share by simultaneously affecting both that party's electoral popularity and the voters' disagreement regarding which party is more desirable. In the next sections, we show that how these two mechanisms interact, together with whether a party is the minority or the majority party, determines the equilibrium issue-selection strategy of a party.

3 Electoral Advantage and Issue Selection

We first investigate whether parties have incentives to advertise an issue on which the opponent has electoral advantage or an issue on which neither party has electoral advantage. In his influential analysis of the strategy of campaigns, Riker (1996) argues that a party does not advertise an issue on which the opponent has advantage (the dominance principle) and that if neither of the two parties has advantage on an issue, both parties ignore that issue (the dispersion principle). Hence, one might think that, for example, if party B advertises an issue i on which party A has electoral advantage (i.e., $\nu_i > 0$), such a strategy is detrimental because advertising an issue on which the voters favor the opponent can only increase the electoral popularity of the opponent. Likewise, one might think that advertising an issue on which neither of the parties has electoral advantage (i.e., $\nu_i = 0$) is not optimal because such advertisement has no effect on a party's electoral popularity but it is costly.

As mentioned, this reasoning captures only one of the effects of promoting an issue. If a party advertises an issue on which its opponent has electoral advantage, such advertisement increases the opponent's vote share by increasing its electoral popularity but it also affects the opponent's vote share by affecting the voters' disagreement regarding which party is more desirable. If the effect of such strategy on the voters' disagreement regarding which party is more desirable is such that it increases the opponent's vote share, then the two mechanisms work in the same direction. However, if advertising an issue on which the opponent has electoral advantage affects the voters' disagreement in a manner that leads to a decrease in the opponent's vote share, then the two mechanisms work in opposite directions. In this section, we show that the two mechanisms work in the same direction for the majority party but in the opposite direction for the minority party. This implies that there can be situations in which the minority party finds it optimal to increase the electoral popularity of its opponent, by advertising an issue on which its opponent has electoral advantage, if such a strategy changes the voters' disagreement regarding which party is more desirable in such a manner that the overall effect is to decrease the opponent's vote share. To show these incentives of the minority party consider the following example.[15]

Example 1. Suppose that there are three policy issues, party A has electoral advantage on issues 1 and 2 and party B has electoral advantage on issue 3. Let $\nu_1 = 10$, $\nu_2 = 1$, $\nu_3 = -2$, $\lambda_{11} = 1$, $\lambda_{22} = 100$, $\lambda_{33} = 1$ and let the weight function be $w_i(\mathbf{a}) = \frac{a_i+1}{\sum_i a_i+3}$. For simplicity, let the action space of each party be binary, $a_i \in \{0, 1\}$ and the cost of advertising an issue be $c(0) = 0$ and $c(1) = 0.05$. Given these specifications, there is an equilibrium in which party B advertises on issue 2 and 3 ($\mathbf{a}^B = (0, 1, 1)$) and party A advertises on issue 1

[15] Note that the purpose of this example is to show the mechanism at work using simple numbers rather then constructing realistic numbers descriptive of any particular electoral contest. A similar rationale applies to all examples presented in this paper.

($\mathbf{a}^A = (1, 0, 0)$). In this equilibrium, the weights of the three issues are equal, $w_i = 1/3$, the vote share of party A is $\Phi(\frac{9}{\sqrt{102}}) \simeq 0.81$, and the vote share of party B is $1 - \Phi(\frac{9}{\sqrt{102}}) \simeq 0.19$; thus party B is the minority party and party A is the majority party. Party B does not have a profitable deviation to $a_2 = 0$ because if party B were not to advertise issue 2, the weights on the three issues would be $w_1 = 2/5$, $w_2 = 1/5$, $w_3 = 2/5$, party A's vote share would be $\Phi(\frac{17}{\sqrt{108}}) \simeq 0.95$, and party B vote share would be $1 - \Phi(\frac{17}{\sqrt{108}}) \simeq 0.05$.

The previous example is set up as minimally as possible to illustrate the minority party's incentive to advertise an issue on which the opponent has electoral advantage. In particular, we restrict the strategy of a party to a discrete choice of advertisement $a_k \in \{0, 1\}$, which implies that party B cannot spend more than 1 unit of advertisement on any issue. One might conjecture that, if given the option, party B might find it beneficial, for example, to allocate the unit of advertisement spent on issue 2 to issue 3 instead. This might be the case because party B has advantage on issue 3 and because, having already spent a unit of advertisement on issue 3, issue advertisement is perhaps more effective the more focus it is. To show that party B's incentives to advertise an issue on which its opponent has electoral advantage are robust to such considerations, let us analyze a modified version of example 1.

Example 2. All parameters are as in example 1 except that party B can allocate the 1 unit of advertisement spent on issue 2 (in example 1) on issue 3 instead and the extra unit of advertisement spent on issue 3 would cost 0 but if party B spends it on advertising issue 2, the cost is 0.05. In this scenario, the allocation of advertisement are $a_1 = 1; a_2 = 0; a_3 = 2$ and the weights are $w_1 = 2/6$, $w_2 = 1/6$, and $w_3 = 3/6$. Party A's vote share is $\Phi(\frac{15}{\sqrt{113}}) \simeq 0.92$ and party B's vote share is $1 - \Phi(\frac{15}{\sqrt{113}}) \simeq 0.08$. Thus party B is worse off to allocate the (extra) unit of advertisement to issue 3 rather than to issue 2 even if it is costless to allocate the unit of advertisement on issue 3 on which party B has an electoral advantage rather than to issue 2 on which party A holds electoral advantage.

The previous example shows that the result that a party may advertise an issue on which its opponent has electoral advantage does not depend on the relative cost of advertising an issue on which that respective party has electoral advantage versus an issue on which the opponent has electoral advantage. Spending a marginal unit of advertisement on an issue on which the opponent has electoral advantage is costly because of the increased electoral popularity of the opponent and because of the opportunity cost of such advertisement (which could be high or low, depending on the assumption on the cost function and what issues on which the minority party has electoral advantage are available). However, these costs are balanced against the benefits of changing the voters' disagreement regarding which party is more desirable, and such benefits depend on the magnitude of λ_{ii}. Fixing the aforementioned costs, if λ_{ii} is big enough, the minority party finds it beneficial to advertise an issue i on which the opponent has electoral advantage. By a similar logic, the minority party can also find it beneficial to advertise issues on which neither of the parties has electoral advantage. To show this incentive consider the following example.

Example 3. Suppose that there are three issues, party A has electoral advantage on issue 1, party B has electoral advantage on issue 3, and neither of the parties has advantage on issue 2. Let $\nu_1 = 10$, $\nu_2 = 0$, $\nu_3 = -2$, $\lambda_{11} = 1$, $\lambda_{22} = 100$, $\lambda_{33} = 1$ and the weight function be $w_i(\mathbf{a}) = \frac{a_i+1}{\sum_i a_i+3}$. Also, let the action space of each party be binary, $a_i \in \{0, 1\}$ and the cost of advertising an issue be $c(0) = 0$ and $c(1) = 0.05$. Given these specifications, there is an equilibrium in which party B advertises on issue 2 and 3 ($\mathbf{a}^B = (0, 1, 1)$) and party A advertises on issue 1 ($\mathbf{a}^A = (1, 0, 0)$).

More generally, the minority party's vote share increases in the advertisement on an issue on which neither of the parties has electoral advantage. Proposition 2 shows that in any equilibrium the minority party advertises an issue on which neither of parties has electoral advantage.

Proposition 2. *The minority party always advertises an issue on which neither party has electoral advantage (that is, if k is the minority party, then $a_i^{k*} > 0$ for all i such that $\nu_i = 0$*

and $\lambda_{ii} > 0$*).*

The previous examples and Proposition 2 show that the minority party can have incentives to advertise an issue on which its opponent has electoral advantage or on which neither party has electoral advantage. On the other hand, the majority party won't advertise such issues in equilibrium when voters' positions across issues are uncorrelated. We have the following result:

Proposition 3. *The majority party does not advertise an issue on which its opponent has electoral advantage or on which neither party has electoral advantage (that is, if party* k *is the majority party, then* $a_i^{k*} = 0$ *for all* i *such that* $\nu_i^k \leq 0$*).*

The intuition of Proposition 3 is that by advertising, for example, an issue on which the minority party has electoral advantage, the majority party will decrease its electoral popularity and, at the same time, increase the voters' disagreement regarding which party is more desirable, both effects working in the same direction to decrease the majority party's vote share. By a similar logic, the majority party will not have incentives to advertise an issue on which neither party has an electoral advantage.

4 Ideological Heterogeneity and Issue Selection

In this section, we assess the counterfactuals of which issues a party is more likely to emphasize on its electoral agenda if that party were to decide between two issues that differ only in terms of the party's electoral advantage ($\nu_i^k > \nu_j^k$ and $\lambda_{ii} = \lambda_{jj}$) or between two issues that differ only in terms of the voters' disagreement regarding which party is more desirable ($\lambda_{ii} > \lambda_{jj}$ and $\nu_i^k = \nu_j^k$).

For one, a party prefers to emphasize more the issue on which it has a bigger electoral advantage if it were to choose between two issues that only differ in terms of a party's electoral advantage, i.e., $\nu_i^k > \nu_j^k$ and $\lambda_{ii} = \lambda_{jj}$. To see the logic of this result note that, when $\nu_i > \nu_j$ and $\lambda_{ii} = \lambda_{jj}$, if party A were to advertise more issue j than i (i.e., $a_j^{A*} > a_i^{A*}$), then party A would have a simple profitable deviation by switching the advertisements on i and j (while keeping the other advertisements the same). Such a strategy doesn't change the relative salience of issues other than i and j, the cost of advertisement or the voters' disagreement regarding which party is more desirable (because $\lambda_{ii} = \lambda_{jj}$) but increases party A's electoral popularity because $\nu_i > \nu_j$. And because party B's vote share is 1 minus the vote share of party A, a similar reasoning applies to why party B prefers to advertise issue i over issue j when $\nu_i < \nu_j$ and $\lambda_{ii} = \lambda_{jj}$. Thus, we have the following result:

Proposition 4. *For any $k \in \{A, B\}$ and $i, j \in I_k$, if $\nu_i^k > \nu_j^k$ and $\lambda_{ii} = \lambda_{jj}$, then $a_i^{k*} \geq a_j^{k*}$.*

The next result indicates that a party's equilibrium status determines whether a party advertises an issue with higher or lower voters' disagreement regarding which party is more desirable. The intuition is as follows. Suppose that party A is the majority party (i.e., $\sum_{i=1}^{n} w_i(\mathbf{a}^*)\nu_i > 0$) and consider its incentive to advertise issue i or issue j when $\lambda_{ii} > \lambda_{jj}$ and $\nu_i = \nu_j$. If party A were to advertise more issue i than issue j ($a_i^* > a_j^*$), party A would have a profitable deviation. It can switch the advertisements on issue i and j; such strategy doesn't change the relative salience of issues other than i and j, the cost of advertisement or party A's electoral popularity (because $\nu_i = \nu_j$) but decreases the voters' disagreement regarding which party is more desirable because $\lambda_{ii} > \lambda_{jj}$, and thus increases party A's vote share. On the other hand, the minority party has the opposite incentive: to advertise the issue with higher voters' disagreement (i.e., issue i) so as to increase the voters' disagreement regarding which party is more desirable on the n policy issues, which, in turn, increases the minority party's vote share. As such, between two issues that differ only in terms of voters' disagreement regarding which party is more desirable, the majority (minority) party advertises more the issue with lower (higher) voters' disagreement. We have the following result:

Proposition 5. *For any $k \in \{A, B\}$ and any $i, j \in I_k$, if $\lambda_{ii} > \lambda_{jj}$ and $\nu_i = \nu_j$, then $a_i^{k*} \leq a_j^{k*}$ if party k is the majority party and $a_i^{k*} \geq a_j^{k*}$ if party k is the minority party.*

Note that Proposition 5 does not imply that a majority (minority) party won't put any positive advertisement on a policy issue with higher (lower) voters' disagreement if other issues with lower (higher) voters' disagreement regarding which party is more desirable (and the same electoral advantage) are available. The majority (minority) party might find it beneficial to advertise an issue with higher (lower) voters' disagreement depending on the configuration of equilibrium issue advertisement, as detailed below. What the proposition suggests is that, in any equilibrium, the majority party advertises more the issue with lower voters' disagreement (within the set of issues that the majority party advertises) and the minority party advertises more the issues with higher voters' disagreement regarding which party is more desirable (within the set of issues that the minority party advertises), all else equal.

As mentioned, the parameter σ_{ii} represents a measure of the ideological heterogeneity of the electorate on issue i since a lower (higher) σ_{ii} implies that voters' policy preferences are more concentrated (dispersed) on issue i. In other words, if we were to compare two policy issues i and j with $\sigma_{ii} > \sigma_{jj}$, all else equal, then policy issue i is more ideologically heterogenous (electorally speaking) than policy issue j. Given that $\lambda_{ii} = (p_i^A - p_i^B)^2\sigma_{ii}$, we have the following result:

Corollary 1. *For any $k \in \{A, B\}$ and any $i, j \in I_k$, if $\sigma_{ii} > \sigma_{jj}$, party k prefers to advertise issue i if it is the minority party and prefers to advertise issue j if it is the majority party, all else equal.*

Corollary 1 follows from the fact that λ_{ii} is increasing in σ_{ii}, and the results regarding the majority and minority parties' equilibrium incentives as stated in Proposition 5. It suggests that the majority party prefers to advertise more issues on which voters are ideologically homogenous while the minority party prefers to advertise more issues on which voters are ideologically heterogenous, all else equal.

As mentioned, Propositions 4 and 5 can be helpful in explaining different kinds of counterfactuals, depending on the type of equilibrium that might result for a specific configuration of parameter values. To illustrate different types of equilibria that may arise, consider the following examples.

Example 4. Suppose that there are 4 policy issues; party B has electoral advantage on issues 1 and 2; and party A has electoral advantage on issues 3 and 4. Let $\nu_1 = \nu_2 = -1$, $\nu_3 = \nu_4 = 2$, $\lambda_{11} = \lambda_{33} = 1$, $\lambda_{22} = \lambda_{44} = 10$, and the weight function be $w_i(\mathbf{a}) = \frac{a_i+1}{\sum_i a_i+4}$. For simplicity, each party can choose among three different levels of advertising on each issue $a_i^k \in \{0, 1, 2\}$ for $k \in \{A, B\}$. The costs of advertisement is as follows: $c^A(2) = 0.05$, $c^A(1) = 0.01$, $c^B(2) = 0.05$ and $c^B(1) = 0.01$. Given these specifications, we have an equilibrium in which party B's strategy is $a_1 = 1$ and $a_2 = 2$ while party A's strategy is $a_3 = 2$ and $a_4 = 1$.

In the equilibrium depicted in example 4, the majority party advertises both the issue with low and high voters' disagreement but emphasizes more the issue with lower voters' disagreement (i.e., issue 3). The minority party advertises both the issue with low and high voters' disagreement regarding which party is more desirable, however, it advertises more the issue with higher voters' disagreement (i.e., issue 2). In this context, Proposition 5 can provide an explanation for why we observe a specific distribution of advertisement across the different issues a party emphasizes on its election agenda.

Example 5. Suppose that there are 4 policy issues, party A has electoral advantage on issues 3 and 4, and party B has electoral advantage on issues 1 and 2. Let $\nu_1 = \nu_2 = -1$, $\nu_3 = \nu_4 = 2$, $\lambda_{11} = \lambda_{33} = 1$, $\lambda_{22} = \lambda_{44} = 10$ and the weight function be $w_i(\mathbf{a}) = \frac{a_i+1}{\sum_i a_i+4}$. For simplicity, suppose that each party can choose among two different levels of advertisement on each issue $a_i^k \in \{0, 1\}$ for $k \in \{A, B\}$. The costs of advertisement is $c^A(1) = 0.07$ and $c^B(1) = 0.07$. There is an equilibrium in which party B advertises issue 2 (i.e., $\mathbf{a}^B = (0, 1, 0, 0)$) and party A advertises issue 3 (i.e., $\mathbf{a}^A = (0, 0, 1, 0)$).

In the equilibrium depicted in example 5, the majority party, party A, only advertises issue 3, the issue with lower voters' disagreement regarding which party is more desirable.

In contrast, the minority party, party B, only advertises issue 2, the issue which has higher voters' disagreement regarding which party is more desirable. In this context, Proposition 5 can provide an explanation for why parties emphasize certain issues and not others on their election agenda, even if those issues do not differ in terms of a party's electoral advantage.

Proposition 5 characterizes the distribution of advertisement *among the issues* that are advertised by a party relative to the other issues on that party's electoral agenda; it does not compare the pattern of issue advertisement across parties. That is, Proposition 5 does not imply that the issues on the majority party's electoral agenda are less ideological heterogeneous than the issues on the minority party's electoral agenda since it does not characterize the pattern of advertisement of one party relative to the other party. To illustrate this point consider the following example:

Example 6. Suppose that there are 3 policy issues, party A has electoral advantage on issues 1, and party B has electoral advantage on issues 2 and 3. Let $\nu_1 = 3$, $\nu_2 = \nu_3 = -1$, $\lambda_{11} = 10$, $\lambda_{22} = 7$, $\lambda_{33} = 1$ and the weight function be $w_i(\mathbf{a}) = \frac{a_i+1}{\sum_i a_i+3}$. For simplicity, suppose that each party can choose among two different levels of advertisement on each issue $a_i^k \in \{0,1\}$ for $k \in \{A,B\}$. The costs of advertisement is $c^A(1) = 0.06$ and $c^B(1) = 0.06$. There is an equilibrium in which party B advertises issue 2 (i.e., $\mathbf{a}^B = (0,1,0)$) and party A advertises issue 1 (i.e., $\mathbf{a}^A = (1,0,0)$) (and note that $\lambda_{22} > \lambda_{33}$ in this equilibrium).

In the Supplementary Appendix, we also analyze the issue-selection incentives of parties in the case in which voters' policy positions across various issues are correlated to show that our results are robust to this extension. Furthermore, such an analysis is also of substantive interest as it allows us to investigate what issues a party is likely to bundle together on its electoral agenda so as to win a higher vote share. We show that the minority party has incentives to advertise more issues with stronger correlations, while the majority party prefers to advertise more issues with weaker correlations, if parties were to choose among issues positively correlated across policy domains. We also show that the minority party has incentives to advertise more issues with weaker correlation while the majority party prefers to advertise issues with stronger correlation, if parties were to choose among issues negatively correlated across policy domains.

5 Conclusion

In this paper, we develop a multidimensional model of electoral competition to investigate the incentives of candidates regarding which positional issues to emphasize during electoral campaigns. The analysis uncovers a novel mechanism by which increasing the salience of policy issues affects a party's vote share, a mechanism that allows us to provide new theoretical results regarding the issue-selection strategy of parties in electoral contests. We show that the minority party has incentives to advertise an issue on which the opponent has electoral advantage or an issue on which neither party has electoral advantage. We also show that the minority party has incentives to emphasize on its electoral agenda issues on which voters are ideologically heterogenous, whereas the majority party has the opposite incentives: it prefers to emphasize on its electoral agenda issues on which voters are ideologically homogenous.

This theoretical analysis provides a foundation for moving toward a more complete understanding on the content of campaign communication in the context of positional issues by highlighting an important determinant of a party's strategic calculus that has gone largely unappreciated to this point: the voters' disagreement regarding which party is more desirable.

Furthermore, the analysis has several empirical implications for the study of electoral campaigns. For one, the analysis suggests that the minority party is more likely than the majority party to advertise positional issues on which the opponent has electoral advantage (i.e., a majority of voters prefer the opponent's policy position on that issue). Another implication of the analysis is that issues on which there is less ideological heterogeneity among voters are more likely to be advertised as compared to issues on which there is less ideological heterogeneity between parties. Moreover, the analysis suggests a certain pattern of campaign advertisement on positional issues that vary in terms of the ideological heterogeneity of voters: the minority party is more likely to campaign on ideologically heterogenous electoral issues (within the set of positional issues on which the minority party campaigns) and the majority party is more likely to emphasize ideologically homogenous electoral issues (within the set of positional issues on the majority party's electoral agenda) during electoral contests. Overall, the analysis provides novel empirical predictions about how the structure of public opinion impacts the positional issue-selection strategy of candidates, which can foster further empirical research on electoral campaigns.

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Appendix

Proof of Proposition 1. Suppose to the contrary that there exists a pure strategy Nash equilibrium $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ such that for some issue i, $a_i^{A*} > 0$ and $a_i^{B*} > 0$. Let $a_i^* = a_i^{A*} + a_i^{B*}$ denote the equilibrium total advertisement on issue i.

Party A's vote share is a function of the total advertisement each issue receives, namely $v^A(\mathbf{a}^A, \mathbf{a}^B) = v^A(a_1, a_2, ..., a_n)$, where $a_i = a_i^A + a_i^B$ denotes the total advertisement on issue i for $i = 1, 2, ..., n$. Similarly, party B's vote share is a function of the total advertisement each issue receives. Because there are only two parties, $v^B(a_1, a_2, ..., a_n) = 1 - v^A(a_1, a_2, ..., a_n)$.

Since $a_i^{A*} > 0$, A's maximization problem implies that $\frac{\partial v^A(a_1^*, ..., a_i^{A*} + a_i^{B*}, ..., a_n^*)}{\partial a_i^A} - c^{A'}(a_i^{A*}) = 0$, and since $\frac{\partial v^A(a_1^*, ..., a_i^{A*} + a_i^{B*}, ..., a_n^*)}{\partial a_i^A} = \frac{\partial v^A(a_1^*, ..., a_i^*, ..., a_n^*)}{\partial a_i}$, we have $\frac{\partial v^A(a_1^*, ..., a_i^*, ..., a_n^*)}{\partial a_i} = c^{A'}(a_i^{A*}) > 0$.

Since $v^B(a_1^*, ..., a_i^*, ..., a_n^*) = 1 - v^A(a_1^*, ..., a_i^*, ..., a_n^*)$, we have $\frac{\partial v^B(a_1^*, ..., a_i^*, ..., a_n^*)}{\partial a_i} = -\frac{\partial v^A(a_1^*, ..., a_i^*, ..., a_n^*)}{\partial a_i}$. Furthermore $\frac{\partial v^B(a_1^*, ..., a_i^{A*} + a_i^{B*}, ..., a_n^*)}{\partial a_i^B} = \frac{\partial v^B(a_1^*, ..., a_i^*, ..., a_n^*)}{\partial a_i}$, so $\frac{\partial v^B(a_1^*, ..., a_i^{A*} + a_i^{B*}, ..., a_n^*)}{\partial a_i^B} = -\frac{\partial v^A(a_1^*, ..., a_i^*, ..., a_n^*)}{\partial a_i} < 0$.

Therefore $\frac{\partial v^B(a_1^*, ..., a_i^{A*} + a_i^{B*}, ..., a_n^*)}{\partial a_i^B} - c^{B'}(a_i^{B*}) < 0$. That is, party B's objective function is strictly decreasing in a_i^B at $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$. Since $a_i^{B*} > 0$, then a_i^{B*} is not the optimal choice for party B and thus we have a contradiction. $\square$

Proof of Proposition 2. Without loss of generality let A be the majority party and B be the minority party. Suppose that there is an equilibrium $\mathbf{a}^*$ in which an issue on which neither party has electoral advantage is not advertised by the minority party. That is, suppose that for some issue i, $\nu_i = 0$ and $a_i^{B*} = 0$.

By Proposition 3 (proved below), we know that $a_i^{A*} = 0$. Then the total amount of advertisement on issue i is $a_i^* = a_i^{A*} + a_i^{B*} = 0$. The equilibrium vote share of party B is

$$v^B(\mathbf{a}^*) = 1 - \Phi\left(\frac{f(a_1^*)\nu_1 + f(a_2^*)\nu_2 + ... + f(a_i^{A*} + a_i^{B*})\nu_i + ... + f(a_n^*)\nu_n}{([f(a_1^*)]^2\lambda_{11} + [f(a_2^*)]^2\lambda_{22} + ... + [f(a_i^{A*} + a_i^{B*})]^2\lambda_{ii} + ... + [f(a_n^*)]^2\lambda_{nn})^{\frac{1}{2}}}\right).$$

$$= 1 - \Phi\left(\frac{f(a_1^*)\nu_1 + f(a_2^*)\nu_2 + ... + f(0)\nu_i + ... + f(a_n^*)\nu_n}{([f(a_1^*)]^2\lambda_{11} + [f(a_2^*)]^2\lambda_{22} + ... + [f(0)]^2\lambda_{ii} + ... + [f(a_n^*)]^2\lambda_{nn})^{\frac{1}{2}}}\right).$$

The first-order derivative of B's vote share with respect to a_i^{B*} at 0 is

$$\frac{\partial}{\partial a_i^{B*}} \upsilon^B(\mathbf{a}^*)_{|a_i^{B*}=0} = \frac{\phi(\frac{\bar{\nu}}{\bar{\lambda}^{\frac{1}{2}}}) f'(0) f(0) \lambda_{ii} \bar{\nu}}{\bar{\lambda}^{\frac{3}{2}}},$$

where $\phi(\cdot)$ is the standard normal pdf, and we denote for simplicity by $\bar{\nu} \equiv \sum_{j=1}^{n}[f(a_j^*)\nu_j]$ and by $\bar{\lambda} \equiv \sum_{j=1}^{n}[f(a_j^*)^2\lambda_{jj}]$.

Since $\phi(\frac{\bar{\nu}}{\bar{\lambda}^{\frac{1}{2}}}) > 0$, $f'(0) > 0$, $f(0) > 0$, $\lambda_{ii} > 0$ and $\bar{\lambda}^{\frac{3}{2}} > 0$, also, $\bar{\nu} > 0$ because A is the majority party.

Therefore, $\frac{\partial}{\partial a_i^{B*}} \upsilon^B(\mathbf{a}^*)_{|a_i^{B*}=0} > 0$. Also we have $c^{B'}(0) = 0$, therefore $\frac{\partial}{\partial a_i^{B*}} \upsilon^B(\mathbf{a}^*)_{|a_i^{B*}=0} - c^{B'}(0) > 0$. That is, party B's objective function is strictly increasing in a_i^{B*} at 0. Hence $\mathbf{a}^*$ cannot be an equilibrium and we have a contradiction.

$\square$

Proof of Proposition 3. Let A be the majority party. We show the result for the case $\nu_i < 0$ and the reasoning for the case $\nu_i = 0$ is similar. Suppose that there is an equilibrium $\mathbf{a}^*$ in which the majority party advertises an issue on which party B is has electoral advantage. That is, suppose that for some issue i, $\nu_i < 0$ and $a_i^{A*} > 0$. By Proposition 1, it follows that $a_i^{B*} = 0$, and thus the total amount of advertisement on issue i is $a_i^* = a_i^{A*}$. Given that the weight function is $w_i(\mathbf{a}) = \frac{f(a_i)}{\sum_{j=1}^{n} f(a_j)}$, the equilibrium vote share of party A is

$$\Phi(\frac{f(a_1^*)\nu_1 + f(a_2^*)\nu_2 + ... + f(a_i^{A*})\nu_i + ... + f(a_n^*)\nu_n}{([f(a_1^*)]^2\lambda_{11} + [f(a_2^*)]^2\lambda_{22} + ... + [f(a_i^{A*})]^2\lambda_{ii} + ... + [f(a_n^*)]^2\lambda_{nn})^{\frac{1}{2}}}).$$

But party A would have a profitable deviation to $a_i^{A'} = 0$. In this case the total amount of advertisement on issue i is $a_i^{'} = a_i^{A'} = 0$. A's vote share by choosing $a_i^{A'} = 0$ would be

$$\Phi(\frac{f(a_1^*)\nu_1 + f(a_2^*)\nu_2 + ... + f(0)\nu_i + ... + f(a_n^*)\nu_n}{([f(a_1^*)]^2\lambda_{11} + [f(a_2^*)]^2\lambda_{22} + ... + [f(0)]^2\lambda_{ii} + ... + [f(a_n^*)]^2\lambda_{nn})^{\frac{1}{2}}}).$$

Because $f(\cdot)$ is increasing and $\nu_i < 0$, then $f(a_1^*)\nu_1 + f(a_2^*)\nu_2 + ... + f(0)\nu_i + ... + f(a_n^*)\nu_n > f(a_1^*)\nu_1 + f(a_2^*)\nu_2 + ... + f(a_i^{A*})\nu_i + ... + f(a_n^*)\nu_n$. Also, $0 < ([f(a_1^*)]^2\lambda_{11} + [f(a_2^*)]^2\lambda_{22} + ... +$

$[f(0)]^2\lambda_{ii}+...+[f(a_n^*)]^2\lambda_{nn})^{\frac{1}{2}} < ([f(a_1^*)]^2\lambda_{11}+[f(a_2^*)]^2\lambda_{22}+...+[f(a_i^{A*})]^2\lambda_{ii}+...+[f(a_n^*)]^2\lambda_{nn})^{\frac{1}{2}}$. As a result, party A's vote share will increase if it deviates to $a_i^{A'} = 0$.

Furthermore, $c^A(a_i^{A*}) > c^A(0)$, hence party A's payoff (vote share minus cost) is higher when choosing $a_i^{A'} = 0$ than choosing a_i^{A*}. Therefore there cannot be an equilibrium in which the majority party advertises an issue on which the other party has electoral advantage. □

Proof of Proposition 4. We will prove the result for party A; the case for party B is analogous.

Let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose that there exist issues i and j such that $\nu_i^A > \nu_j^A$ (i.e. $\nu_i > \nu_j$) and $\lambda_{ii} = \lambda_{jj}$, but to the contrary of our proposition that $a_i^{A*} < a_j^{A*}$. Since $i, j \in I_A$, $a_i^{B*} = a_j^{B*} = 0$, so $a_i^* = a_i^{A*}$ and $a_j^* = a_j^{A*}$.

Now consider a different advertisement vector $\mathbf{a}^{A'}$ defined by $a_i^{A'} = a_j^{A*}$, $a_j^{A'} = a_i^{A*}$, and $a_l^{A'} = a_l^{A*}$ for all $l \neq i, j$. Let $\mathbf{a}' = (\mathbf{a}^{A'}, \mathbf{a}^{B*})$.

Since $a_i^* = a_i^{A*} < a_j^{A*} = a_j^*$, and $f(\cdot)$ is increasing, we have $w_i(\mathbf{a}^*) < w_j(\mathbf{a}^*)$. Denote $\mathbf{w}' = \mathbf{w}(\mathbf{a}')$ and $\mathbf{w}^* = \mathbf{w}(\mathbf{a}^*)$, that is $w_i^* < w_j^*$. Since $w_i(\mathbf{a}) = \frac{f(a_i)}{\sum_{i=1}^n f(a_i)}$, we also have $w_i' = w_j^*$, $w_j' = w_i^*$, and $w_l' = w_l^*$ for all $l \neq i, j$.

Therefore

$$\begin{aligned}\sum_{l=1}^n w_l'\nu_l - \sum_{l=1}^n w_l^*\nu_l &= (w_i' - w_i^*)\nu_i + (w_j' - w_j^*)\nu_j \\ &= (w_j^* - w_i^*)(\nu_i - \nu_j) > 0.\end{aligned}$$

That is, $\sum_{l=1}^n w_l'\nu_l > \sum_{l=1}^n w_l^*\nu_l$.

And similarly,

$$\begin{aligned}\sum_{l=1}^n w_l'^2\lambda_{ll} - \sum_{l=1}^n w_l^{*2}\lambda_{ll} &= (w_i'^2 - w_i^{*2})\lambda_{ii} + (w_j'^2 - w_j^{*2})\lambda_{jj} \\ &= (w_j^{*2} - w_i^{*2})(\lambda_{ii} - \lambda_{jj}) = 0,\end{aligned}$$

because $\lambda_{ii} = \lambda_{jj}$. That is, $\sum_{l=1}^n w_l'^2\lambda_{ll} = \sum_{l=1}^n w_l^{*2}\lambda_{ll} > 0$.

Furthermore, from the construction of $\mathbf{a}^{A'}$, $\sum_{l=1}^{n} c^A(a_l^{A'}) = \sum_{l=1}^{n} c^A(a_l^{A*})$.

Therefore,

$$\Phi\left(\frac{\sum_{l=1}^{n} w_l^{'}\nu_l}{(\sum_{l=1}^{n} w_l^{'2}\lambda_{ll})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^{n} w_l^{*}\nu_l}{(\sum_{l=1}^{n} w_l^{*2}\lambda_{ll})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A*}).$$

Therefore $\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, a contradiction. □

Proof of Proposition 5. We will prove the result for party A; the case for party B is analogous.

First, suppose A is the majority party, and let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose that there exist issues i and j such that $\lambda_{ii} > \lambda_{jj}$ and $\nu_i = \nu_j$, but to the contrary of our proposition that $a_i^{A*} > a_j^{A*}$. Since $i, j \in I_A$, $a_i^{B*} = a_j^{B*} = 0$, so $a_i^* = a_i^{A*}$ and $a_j^* = a_j^{A*}$.

Now consider a different advertisement vector $\mathbf{a}^{A'}$ defined by $a_i^{A'} = a_j^{A*}$, $a_j^{A'} = a_i^{A*}$, and $a_l^{A'} = a_l^{A*}$ for all $l \neq i, j$. Let $\mathbf{a}^{'} = (\mathbf{a}^{A'}, \mathbf{a}^{B*})$.

Since $a_i^* = a_i^{A*} > a_j^{A*} = a_j^*$, and $f(\cdot)$ is increasing, we have $w_i(\mathbf{a}^*) > w_j(\mathbf{a}^*)$. Denote $\mathbf{w}^{'} = \mathbf{w}(\mathbf{a}^{'})$ and $\mathbf{w}^* = \mathbf{w}(\mathbf{a}^*)$, that is $w_i^* > w_j^*$. Since $w_i(\mathbf{a}) = \frac{f(a_i)}{\sum_{i=1}^{n} f(a_i)}$, we also have $w_i^{'} = w_j^*$, $w_j^{'} = w_i^*$, and $w_l^{'} = w_l^*$ for all $l \neq i, j$.

Therefore

$$\begin{aligned}\sum_{l=1}^{n} w_l^{'}\nu_l - \sum_{l=1}^{n} w_l^{*}\nu_l &= (w_i^{'} - w_i^*)\nu_i + (w_j^{'} - w_j^*)\nu_j \\ &= (w_j^* - w_i^*)(\nu_i - \nu_j) = 0,\end{aligned}$$

because $\nu_i = \nu_j$. That is, $\sum_{l=1}^{n} w_l^{'}\nu_l = \sum_{l=1}^{n} w_l^{*}\nu_l > 0$, where $\sum_{l=1}^{n} w_l^{*}\nu_l > 0$ is because A is the majority party.

Similarly,

$$\begin{aligned}\sum_{l=1}^{n} w_l^{'2}\lambda_{ll} - \sum_{l=1}^{n} w_l^{*2}\lambda_{ll} &= (w_i^{'2} - w_i^{*2})\lambda_{ii} + (w_j^{'2} - w_j^{*2})\lambda_{jj} \\ &= (w_j^{*2} - w_i^{*2})(\lambda_{ii} - \lambda_{jj}) < 0,\end{aligned}$$

because $w_i^* > w_j^*$ and $\lambda_{ii} > \lambda_{jj}$. That is, $0 < \sum_{l=1}^n w_l'^2 \lambda_{ll} < \sum_{l=1}^n w_l^{*2} \lambda_{ll}$.

Furthermore, from the construction of $\mathbf{a}^{A'}$, $\sum_{l=1}^n c^A(a_l^{A'}) = \sum_{l=1}^n c^A(a_l^{A*})$.

Therefore,

$$\Phi\left(\frac{\sum_{l=1}^n w_l' \nu_l}{(\sum_{l=1}^n w_l'^2 \lambda_{ll})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^n w_l^* \nu_l}{(\sum_{l=1}^n w_l^{*2} \lambda_{ll})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A*}).$$

Therefore $\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, a contradiction.

The proof for the case in which A is the minority party is similar to the above. Suppose A is the minority party and let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose there exist two issues $i, j \in I_A$, such that $\lambda_{ii} > \lambda_{jj}$ and $\nu_i = \nu_j$, but to the contrary of our proposition that $a_i^{A*} < a_j^{A*}$.

We can construct the alternative advertisement vector $\mathbf{a}^{A'}$ just like above, and in this case $\sum_{l=1}^n w_l' \nu_l = \sum_{l=1}^n w_l^* \nu_l < 0$ because A is the minority party, $\sum_{l=1}^n w_l'^2 \lambda_{ll} > \sum_{l=1}^n w_l^{*2} \lambda_{ll} > 0$, and $\sum_{l=1}^n c^A(a_l^{A'}) = \sum_{l=1}^n c^A(a_l^{A*})$.

Hence

$$\Phi\left(\frac{\sum_{l=1}^n w_l' \nu_l}{(\sum_{l=1}^n w_l'^2 \lambda_{ll})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^n w_l^* \nu_l}{(\sum_{l=1}^n w_l^{*2} \lambda_{ll})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A*}),$$

$\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, again we have a contradiction. $\square$

Supplementary Appendix: Correlated Issues

Correlated Issues

In this section, we analyze the issue-selection incentives of parties in the case in which voters' policy positions across various issues are correlated to show that the previous results are robust to this extension. Furthermore, such an analysis is also of substantive interest as it allows us to investigate what issues a party is likely to bundle together on its electoral agenda so as to win a higher vote share. The proofs of the results are contained in the next section.

The derivation of voters' optimal decision and the parties' vote share is similar to the analysis of Section 2. Therefore, the electoral popularity of party A from the perspective of a voter with ideal policy $\mathbf{x}$ on the n policy issues is multivariate normal:

$$\mathbf{d}(\mathbf{x}) \equiv (d_1(x_1), d_2(x_2), ..., d_n(x_n))' \sim N(\boldsymbol{\nu}^A, \boldsymbol{\Lambda}),$$

where

$$\nu_i^A = (p_i^A - p_i^B)(\mu_i - \frac{p_i^A + p_i^B}{2})$$

and

$$\lambda_{ij} = (p_i^A - p_i^B)(p_j^A - p_j^B)\sigma_{ij},$$

where λ_{ij} represent the entries of the variance-covariance matrix $\boldsymbol{\Lambda}$. Again, we use the notation ν_i^k for $k \in \{A, B\}$ and, without loss of generality, denote $\nu_i = \nu_i^A$ where indexing by k is not relevant. Party A's vote share is as follows:

$$v^A(\mathbf{a}^A; \mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\mathbf{w}(\mathbf{a}) \cdot \mathbf{d}(\mathbf{x}) > 0) = \Phi\left(\frac{\sum_{i=1}^n w_i(\mathbf{a})\nu_i}{(\sum_{i=1}^n \sum_{j=1}^n w_i(\mathbf{a})w_j(\mathbf{a})\lambda_{ij})^{\frac{1}{2}}}\right), \tag{6}$$

where $\Phi(\cdot)$ is the cdf of standard normal distribution.

Similar to the previous analysis, we can think of ν_i as a measure of party A's electoral popularity on policy issue i. The parameter λ_{ii} can be thought as a measure of the voters' disagreement regarding which party is more desirable on issue i. Finally, the parameter λ_{ij} for $j \neq i$ can be thought as a measure of the correlation of a party's electoral popularity between issue i and issue j. Therefore, the voters' disagreement regarding which party is more desirable on the n policy issues, the denominator of expression (6), consists of the sum of the voters' disagreement regarding which party is more desirable on each issue dimension (i.e. $\sum_{i=1}^n (w_i(\mathbf{a}))^2\lambda_{ii}$) and of the correlations of a party's electoral popularity across various issues (i.e., $\sum_{j\neq i}\sum_{i=1}^{n} w_i(\mathbf{a})w_j(\mathbf{a})\lambda_{ij}$).[16] The salience of each issue dimension, $w_i(\mathbf{a})$, determines how the voters' disagreement and the correlation parameters (λ_{ii} and λ_{ij}) are aggregated across the n policy issues.

The definition of the majority and the minority party as well as the definition of the set of potential issues I_k a party may advertise are defined in a similar manner to the previous analysis.

As mentioned, when issues are correlated, the majority party may find it beneficial to advertise an issue on which the opponent has electoral advantage or an issue on which neither of the parties has electoral advantage. To see this consider the following example.

Example 7. Suppose that there are 3 policy issues, and let $\nu_1 = 2$, $\nu_2 = -1$, $\nu_3 = -0.05$, $\lambda_{11} = \lambda_{22} = \lambda_{33} = 1$, $\lambda_{13} = -1$, $\lambda_{12} = \lambda_{23} = 0$ and the weight function be $w_i(\mathbf{a}) = \frac{a_i+1}{\sum_i a_i+3}$. For simplicity, let the action space of each party be binary, $a_i^k \in \{0,1\}$ and let the cost of advertising an issue be $c^A(0) = c^B(0) = 0$, $c^A(1) = 0.01$, and $c^B(1) = 0.04$. Given these specifications, we have an equilibrium in which party A advertises issues 1 and 3 and party B advertises issue 2. The weights of the three issues are $w_1 = w_2 = w_3 = 1/3$; party A's equilibrium vote share is $\Phi(0.95) \simeq 0.83$ and party B's equilibrium vote share is $1-\Phi(0.95) \simeq 0.17$. If party A were to choose not to advertise issue 3, then the weights of the three issue would be $w_1 = w_2 = 2/5$ and $w_3 = 1/5$; party A's equilibrium vote share would be $\Phi(\frac{1.95}{\sqrt{5}}) \simeq 0.81$ and party B's equilibrium vote share would be $1 - \Phi(\frac{1.95}{\sqrt{5}}) \simeq 0.19$. Thus this example shows that the majority party has incentive to advertise an issue on which the minority party has electoral advantage.

Example 8. All parameters are as in example 6 except that $\nu_3 = 0$. Similar to example 6, we have an equilibrium in which party A advertises issue 1 and 3 and party B advertises issue 2; party A's equilibrium vote share is $\Phi(1) \simeq 0.84$ and party B's equilibrium vote

[16]This is the case because we can re-write the denominator of expression (6), as $(\sum_{i=1}^{n}\sum_{j=1}^{n} w_i(\mathbf{a})w_j(\mathbf{a})\lambda_{ij})^{\frac{1}{2}} = (\sum_{i=1}^{n}(w_i(\mathbf{a}))^2\lambda_{ii} + \sum_{j\neq i}\sum_{i=1}^{n} w_i(\mathbf{a})w_j(\mathbf{a})\lambda_{ij})^{\frac{1}{2}}$.

share is $1 - \Phi(1) \simeq 0.16$. If party A were to choose not to advertise issue 3, then party A's equilibrium vote share would be $\Phi(\frac{2}{\sqrt{5}}) \simeq 0.81$, and therefore the majority party's payoff is lower if deviates to $a_3 = 0$. Thus this example shows that the majority party has incentive to advertise an issue on which neither of the parties has electoral advantage.

These examples, together with the results from Section 2, underscore some limitations on Riker's dominance and dispersion principles. The dominance principle, for example, identifies only one aspect of a party's strategic calculus: advertising on an issue on which the other party has advantage increases the opponent's electoral popularity. Nevertheless, such advertisement strategy can still be beneficial for a party if it changes the voters' disagreement regarding which party is more desirable with the overall effect of increasing that respective party's vote share. With correlated issues, both the majority and the minority party might find advertising an issue on which the opponent has electoral advantage beneficial; the minority party would advertise such issues in order to increase the voters' disagreement regarding which party is more desirable, while the majority party prefers to advertise such issues so as to decrease the voters' disagreement regarding which party is more desirable on the n policy issues. Thus, if it were to advertise issues on which the opponent has electoral advantage, the minority party prefers issues with high variance or with high positive correlations with other issues while the majority party prefers issues with high negative correlations with other issues.

Recall that Propositions 4 and 5 document which issues a party is more likely to advertise if a party were to decide between two issues that only differ in terms of the party's electoral popularity ($\nu_i^k > \nu_j^k$ and $\lambda_{ii} = \lambda_{jj}$) or between two issues that only differ in terms of the voters' disagreement regarding which party is more desirable ($\lambda_{ii} > \lambda_{jj}$ and $\nu_i = \nu_j$) when issue i and j are uncorrelated (i.e., $\sigma_{ij} = 0$). We re-state Propositions 4 and 5 to show that a party's optimal decision whether to advertise more on issue i or issue j holds for any $\sigma_{ij} \neq 0$. We have the following results:

Proposition 4'. For any $k \in \{A, B\}$ and $i, j \in I_k$, if $\nu_i^k > \nu_j^k$, $\lambda_{ii} = \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for $l \neq i, j$, then $a_i^{k*} \geq a_j^{k*}$.

Proposition 5'. For any $k \in \{A, B\}$ and $i, j \in I_k$, if $\lambda_{ii} > \lambda_{jj}$, $\nu_i = \nu_j$ and $\lambda_{il} = \lambda_{jl}$ for $l \neq i, j$, then $a_i^{k*} \geq a_j^{k*}$ if party k is the minority party and $a_i^{k*} \leq a_j^{k*}$ if party k is the majority party.

Similarly, Corollary 1 can be generalized to the situation in which issues are correlated. That is, for any $i, j \in I_k$, if $\sigma_{ii} > \sigma_{jj}$, party k prefers to advertise issue i if it is the minority party and prefers to advertise issue j if it is the majority party, all else equal. This statement follows from Proposition $5'$ given that $\lambda_{ii} = (p_i^A - p_i^B)^2 \sigma_{ii}$.

Furthermore, we can investigate which issue a party is more likely to advertise if it were to decide between two issues i and j that only differ in terms of their correlations with some other issue h, all else equal. We have the following result:

Proposition 6. *For any $k \in \{A, B\}$ and $i, j \in I_k$, if $\lambda_{ih} > \lambda_{jh}$ for some $h \neq i, j$, $\nu_i = \nu_j$, $\lambda_{ii} = \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq h, i, j$, then $a_i^{k*} \geq a_j^{k*}$ if party k is the minority party and $a_i^{k*} \leq a_j^{k*}$ if k is the majority party.*

The intuition of Proposition 6 is as follows. All else equal, the minority party has incentives to advertise more on those issues that increase the voters' disagreement regarding which party is more desirable, and therefore increases the minority party's vote share. The majority party has the opposite incentives: to advertise more on those issues that decrease the voters' disagreement regarding which party is more desirable on the n policy issues so as to increase the majority party's vote share. As a result, if a party were to choose between two issues i and j with $\lambda_{ih} > \lambda_{jh}$, all else equal, the minority party advertises more on issue i while the majority party emphasizes more issue j. A corollary of Proposition 6 is the following: the minority party has incentives to advertise more the issue with stronger correlation, while the majority party prefers to advertise more the issue with weaker correlation, if parties were to choose between two issues i and j that are positively correlated with some other issue h (i.e., $\lambda_{ih} > \lambda_{jh} > 0$), all else equal. Similarly, the minority party prefers to advertise more the issue with weaker correlation while the majority has incentive to advertise more the issue with stronger correlation, if parties were to choose between two issues i and j that are negatively correlated with some other issue h (i.e., $0 > \lambda_{ih} > \lambda_{jh}$), all else equal.

Proofs of Propositions

Proof of Proposition 4'. We will prove the result for party A; the case for party B is analogous.

Let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose to the contrary of our proposition, that there exist two issues $i, j \in I_A$, such that $\nu_i^A > \nu_j^A$ (i.e. $\nu_i > \nu_j$), $\lambda_{ii} = \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq i, j$, but $a_i^{A*} < a_j^{A*}$. Since $i, j \in I_A$, $a_i^{B*} = a_j^{B*} = 0$, so $a_i^* = a_i^{A*}$ and $a_j^* = a_j^{A*}$.

Now consider a different advertisement vector $\mathbf{a}^{A'}$ defined by $a_i^{A'} = a_j^{A*}$, $a_j^{A'} = a_i^{A*}$, and $a_l^{A'} = a_l^{A*}$ for all $l \neq i, j$. Let $\mathbf{a}' = (\mathbf{a}^{A'}, \mathbf{a}^{B*})$.

Since $a_i^* = a_i^{A*} < a_j^{A*} = a_j^*$, and $f(\cdot)$ is increasing, we have $w_i(\mathbf{a}^*) < w_j(\mathbf{a}^*)$. Denote $\mathbf{w}' = \mathbf{w}(\mathbf{a}')$ and $\mathbf{w}^* = \mathbf{w}(\mathbf{a}^*)$, that is $w_i^* < w_j^*$. Since $w_i(\mathbf{a}) = \frac{f(a_i)}{\sum_{i=1}^n f(a_i)}$, we also have $w_i' = w_j^*$, $w_j' = w_i^*$, and $w_l' = w_l^*$ for all $l \neq i, j$.

Therefore

$$\begin{aligned}\sum_{l=1}^n w_l' \nu_l - \sum_{l=1}^n w_l^* \nu_l &= (w_i' - w_i^*)\nu_i + (w_j' - w_j^*)\nu_j \\ &= (w_j^* - w_i^*)(\nu_i - \nu_j) > 0.\end{aligned}$$

That is, $\sum_{l=1}^n w_l' \nu_l > \sum_{l=1}^n w_l^* \nu_l$.

Similarly,

$$\sum_{l=1}^n \sum_{h=1}^n w_l' w_h' \lambda_{lh} - \sum_{l=1}^n \sum_{h=1}^n w_l^* w_h^* \lambda_{lh}$$

$$= (w_i^{'2} - w_i^{*2})\lambda_{ii} + (w_j^{'2} - w_j^{*2})\lambda_{jj} + 2\sum_{l \neq i,j} w_l^* \lambda_{il}(w_i^{'} - w_i^*) + 2\sum_{l \neq i,j} w_l^* \lambda_{jl}(w_j^{'} - w_j^*)$$

$$= (w_j^{*2} - w_i^{*2})\lambda_{ii} + (w_i^{*2} - w_j^{*2})\lambda_{jj} + 2\sum_{l \neq i,j} w_l^* \lambda_{il}(w_j^* - w_i^*) + 2\sum_{l \neq i,j} w_l^* \lambda_{jl}(w_i^* - w_j^*)$$

$$= (w_j^{*2} - w_i^{*2})(\lambda_{ii} - \lambda_{jj}) + 2\sum_{l \neq i,j} w_l^* (w_j^* - w_i^*)(\lambda_{il} - \lambda_{jl})$$

$$= 0,$$

because $\lambda_{ii} = \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq i, j$.

That is, $\sum_{l=1}^n \sum_{h=1}^n w_l^{'} w_h^{'} \lambda_{lh} = \sum_{l=1}^n \sum_{h=1}^n w_l^* w_h^* \lambda_{lh} > 0$.

Furthermore, from the construction of $\mathbf{a}^{A'}$, $\sum_{l=1}^n c^A(a_l^{A'}) = \sum_{l=1}^n c^A(a_l^{A*})$.

Therefore,

$$\Phi\left(\frac{\sum_{l=1}^n w_l^{'}\nu_l}{(\sum_{l=1}^n \sum_{h=1}^n w_l^{'} w_h^{'} \lambda_{lh})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^n w_l^*\nu_l}{(\sum_{l=1}^n \sum_{h=1}^n w_l^* w_h^* \lambda_{lh})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A*}),$$

$\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, a contradiction.

□

Proof of Proposition 5'. We will prove the result for party A; the case for party B is analogous.

First, suppose A is the majority party, and let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose to the contrary of our proposition, that there exist two issues $i, j \in I_A$, such that $\nu_i = \nu_j$, $\lambda_{ii} > \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq i, j$, but $a_i^{A*} > a_j^{A*}$. Since $i, j \in I_A$, $a_i^{B*} = a_j^{B*} = 0$, so $a_i^* = a_i^{A*}$ and $a_j^* = a_j^{A*}$.

Now consider a different advertisement vector $\mathbf{a}^{A'}$ defined by $a_i^{A'} = a_j^{A*}$, $a_j^{A'} = a_i^{A*}$, and $a_l^{A'} = a_l^{A*}$ for all $l \neq i, j$. Let $\mathbf{a}^{'} = (\mathbf{a}^{A'}, \mathbf{a}^{B*})$.

Since $a_i^* = a_i^{A*} > a_j^{A*} = a_j^*$, and $f(\cdot)$ is increasing, we have $w_i(\mathbf{a}^*) > w_j(\mathbf{a}^*)$. Denote $\mathbf{w}^{'} = \mathbf{w}(\mathbf{a}^{'})$ and $\mathbf{w}^* = \mathbf{w}(\mathbf{a}^*)$, that is $w_i^* > w_j^*$. Since $w_i(\mathbf{a}) = \frac{f(a_i)}{\sum_{i=1}^n f(a_i)}$, we also have $w_i^{'} = w_j^*$, $w_j^{'} = w_i^*$, and $w_l^{'} = w_l^*$ for all $l \neq i, j$.

Therefore

$$\sum_{l=1}^{n} w_l^{'}\nu_l - \sum_{l=1}^{n} w_l^{*}\nu_l = (w_i^{'} - w_i^{*})\nu_i + (w_j^{'} - w_j^{*})\nu_j$$
$$= (w_j^{*} - w_i^{*})(\nu_i - \nu_j) = 0.$$

That is, $\sum_{l=1}^{n} w_l^{'}\nu_l = \sum_{l=1}^{n} w_l^{*}\nu_l > 0$, where $\sum_{l=1}^{n} w_l^{*}\nu_l > 0$ is because A is the majority party.

Similarly,

$$\sum_{l=1}^{n}\sum_{h=1}^{n} w_l^{'}w_h^{'}\lambda_{lh} - \sum_{l=1}^{n}\sum_{h=1}^{n} w_l^{*}w_h^{*}\lambda_{lh}$$
$$= (w_i^{'2} - w_i^{*2})\lambda_{ii} + (w_j^{'2} - w_j^{*2})\lambda_{jj} + 2\sum_{l\neq i,j} w_l^{*}\lambda_{il}(w_i^{'} - w_i^{*}) + 2\sum_{l\neq i,j} w_l^{*}\lambda_{jl}(w_j^{'} - w_j^{*})$$
$$= (w_j^{*2} - w_i^{*2})\lambda_{ii} + (w_i^{*2} - w_j^{*2})\lambda_{jj} + 2\sum_{l\neq i,j} w_l^{*}\lambda_{il}(w_j^{*} - w_i^{*}) + 2\sum_{l\neq i,j} w_l^{*}\lambda_{jl}(w_i^{*} - w_j^{*})$$
$$= (w_j^{*2} - w_i^{*2})(\lambda_{ii} - \lambda_{jj}) + 2\sum_{l\neq i,j} w_l^{*}(w_j^{*} - w_i^{*})(\lambda_{il} - \lambda_{jl})$$
$$< 0,$$

because $w_i^{*} > w_j^{*}$, $\lambda_{ii} > \lambda_{jj}$, and $\lambda_{il} = \lambda_{jl}$ for all $l \neq i, j$.

That is, $0 < \sum_{l=1}^{n}\sum_{h=1}^{n} w_l^{'}w_h^{'}\lambda_{lh} < \sum_{l=1}^{n}\sum_{h=1}^{n} w_l^{*}w_h^{*}\lambda_{lh}$.

Furthermore, from the construction of $\mathbf{a}^{A'}$, $\sum_{l=1}^{n} c^A(a_l^{A'}) = \sum_{l=1}^{n} c^A(a_l^{A*})$.

Therefore,

$$\Phi\left(\frac{\sum_{l=1}^{n} w_l^{'}\nu_l}{(\sum_{l=1}^{n}\sum_{h=1}^{n} w_l^{'}w_h^{'}\lambda_{lh})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^{n} w_l^{*}\nu_l}{(\sum_{l=1}^{n}\sum_{h=1}^{n} w_l^{*}w_h^{*}\lambda_{lh})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A*}),$$

$\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, a contradiction.

The proof for the case in which A is the minority party is similar to the above. Suppose

A is the minority party and let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose to the contrary of our proposition, that there exist two issues $i, j \in I_A$, such that $\nu_i = \nu_j$, $\lambda_{ii} > \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq i, j$, but $a_i^{A*} < a_j^{A*}$.

We can construct the alternative advertisement vector $\mathbf{a}^{A'}$ just like above, and in this case $\sum_{l=1}^n w_l^{'} \nu_l = \sum_{l=1}^n w_l^* \nu_l < 0$ because A is the minority party, $\sum_{l=1}^n \sum_{h=1}^n w_l^{'} w_h^{'} \lambda_{lh} > \sum_{l=1}^n \sum_{h=1}^n w_l^* w_h^* \lambda_{lh} > 0$, and $\sum_{l=1}^n c^A(a_l^{A'}) = \sum_{l=1}^n c^A(a_l^{A*})$.

Hence

$$\Phi\left(\frac{\sum_{l=1}^n w_l^{'} \nu_l}{(\sum_{l=1}^n \sum_{h=1}^n w_l^{'} w_h^{'} \lambda_{lh})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^n w_l^* \nu_l}{(\sum_{l=1}^n \sum_{h=1}^n w_l^* w_h^* \lambda_{lh})^{\frac{1}{2}}}\right) - \sum_{l=1}^n c^A(a_l^{A*}),$$

$\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, again we have a contradiction. □

Proof of Proposition 6. We will prove the result for party A; the case for party B is analogous.

First, suppose A is the majority party, and let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose to the contrary of our proposition, that there exist two issues $i, j \in I_A$, such that $\lambda_{ih} > \lambda_{jh}$ for some $h \neq i, j$, $\nu_i = \nu_j$, $\lambda_{ii} = \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq h, i, j$, but $a_i^{A*} > a_j^{A*}$. Since $i, j \in I_A$, $a_i^{B*} = a_j^{B*} = 0$, so $a_i^* = a_i^{A*}$ and $a_j^* = a_j^{A*}$.

Now consider a different advertisement vector $\mathbf{a}^{A'}$ defined by $a_i^{A'} = a_j^{A*}$, $a_j^{A'} = a_i^{A*}$, and $a_l^{A'} = a_l^{A*}$ for all $l \neq i, j$. Let $\mathbf{a}^{'} = (\mathbf{a}^{A'}, \mathbf{a}^{B*})$.

Since $a_i^* = a_i^{A*} > a_j^{A*} = a_j^*$, and $f(\cdot)$ is increasing, we have $w_i(\mathbf{a}^*) > w_j(\mathbf{a}^*)$. Denote $\mathbf{w}^{'} = \mathbf{w}(\mathbf{a}^{'})$ and $\mathbf{w}^* = \mathbf{w}(\mathbf{a}^*)$, that is $w_i^* > w_j^*$. Since $w_i(\mathbf{a}) = \frac{f(a_i)}{\sum_{i=1}^n f(a_i)}$, we also have $w_i^{'} = w_j^*$, $w_j^{'} = w_i^*$, and $w_l^{'} = w_l^*$ for all $l \neq i, j$.

Therefore

$$\begin{aligned}\sum_{l=1}^n w_l^{'} \nu_l - \sum_{l=1}^n w_l^* \nu_l &= (w_i^{'} - w_i^*)\nu_i + (w_j^{'} - w_j^*)\nu_j \\ &= (w_j^* - w_i^*)(\nu_i - \nu_j) = 0.\end{aligned}$$

That is, $\sum_{l=1}^{n} w_l^{'}\nu_l = \sum_{l=1}^{n} w_l^{*}\nu_l > 0$, where $\sum_{l=1}^{n} w_l^{*}\nu_l > 0$ is because A is the majority party.

Similarly,

$$\begin{aligned}
&\sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{'}w_z^{'}\lambda_{lz} - \sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{*}w_z^{*}\lambda_{lz} \\
&= (w_i^{'2} - w_i^{*2})\lambda_{ii} + (w_j^{'2} - w_j^{*2})\lambda_{jj} + 2\sum_{l\neq i,j} w_l^{*}\lambda_{il}(w_i^{'} - w_i^{*}) + 2\sum_{l\neq i,j} w_l^{*}\lambda_{jl}(w_j^{'} - w_j^{*}) \\
&= (w_j^{*2} - w_i^{*2})\lambda_{ii} + (w_i^{*2} - w_j^{*2})\lambda_{jj} + 2\sum_{l\neq i,j} w_l^{*}\lambda_{il}(w_j^{*} - w_i^{*}) + 2\sum_{l\neq i,j} w_l^{*}\lambda_{jl}(w_i^{*} - w_j^{*}) \\
&= (w_j^{*2} - w_i^{*2})(\lambda_{ii} - \lambda_{jj}) + 2\sum_{l\neq i,j} w_l^{*}(w_j^{*} - w_i^{*})(\lambda_{il} - \lambda_{jl}) \\
&= 2w_h^{*}(w_j^{*} - w_i^{*})(\lambda_{ih} - \lambda_{jh}) < 0,
\end{aligned}$$

because $w_i^{*} > w_j^{*}$, $\lambda_{ih} > \lambda_{jh}$, $\lambda_{ii} = \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq h, i, j$.

That is, $0 < \sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{'}w_z^{'}\lambda_{lz} < \sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{*}w_z^{*}\lambda_{lz}$.

Furthermore, from the construction of $\mathbf{a}^{A'}$, $\sum_{l=1}^{n} c^A(a_l^{A'}) = \sum_{l=1}^{n} c^A(a_l^{A*})$.

Therefore,

$$\Phi\left(\frac{\sum_{l=1}^{n} w_l^{'}\nu_l}{(\sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{'}w_z^{'}\lambda_{lz})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^{n} w_l^{*}\nu_l}{(\sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{*}w_z^{*}\lambda_{lz})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A*}),$$

$\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, a contradiction.

The proof for the case in which A is the minority party is similar to the above. Suppose A is the minority party and let $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ be an equilibrium. Suppose to the contrary of our proposition, that there exist two issues $i, j \in I_A$, such that $\lambda_{ih} > \lambda_{jh}$ for some $h \neq i, j$, $\nu_i = \nu_j$, $\lambda_{ii} = \lambda_{jj}$ and $\lambda_{il} = \lambda_{jl}$ for all $l \neq h, i, j$, but $a_i^{A*} < a_j^{A*}$. Since $i, j \in I_A$, $a_i^{B*} = a_j^{B*} = 0$, so $a_i^{*} = a_i^{A*}$ and $a_j^{*} = a_j^{A*}$.

We can construct the alternative advertisement vector $\mathbf{a}^{A'}$ just like above, and in this case $\sum_{l=1}^{n} w_l^{'}\nu_l = \sum_{l=1}^{n} w_l^{*}\nu_l < 0$ because A is the minority party, $\sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{'}w_z^{'}\lambda_{lz} > \sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{*}w_z^{*}\lambda_{lz} > 0$, and $\sum_{l=1}^{n} c^A(a_l^{A'}) = \sum_{l=1}^{n} c^A(a_l^{A*})$.

Hence

$$\Phi\left(\frac{\sum_{l=1}^{n} w_l^{'}\nu_l}{(\sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{'}w_z^{'}\lambda_{lz})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A'}) > \Phi\left(\frac{\sum_{l=1}^{n} w_l^{*}\nu_l}{(\sum_{l=1}^{n}\sum_{z=1}^{n} w_l^{*}w_z^{*}\lambda_{lz})^{\frac{1}{2}}}\right) - \sum_{l=1}^{n} c^A(a_l^{A*}),$$

$\mathbf{a}^{A'}$ is an improvement on $\mathbf{a}^{A*}$, again we have a contradiction. □